# 1.1 Limits Graphically

Name:_____

Notes

What is a limit?

A **limit** is the _____ a function _____ from *both* the left and the right side of a given _____.

Example 1

$f(-3) =$

$\lim\limits_{x \to -3} f(x) =$

$f(-3) =$

$\lim\limits_{x \to -3} f(x) =$

$f(-3) =$
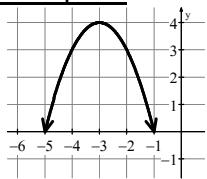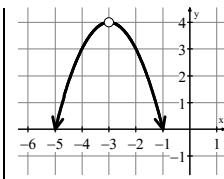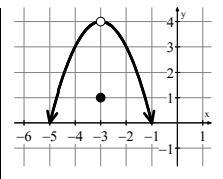
$\lim\limits_{x \to -3} f(x) =$
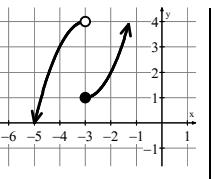
$f(-3) =$
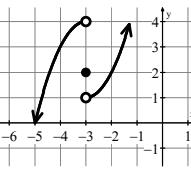
$\lim\limits_{x \to -3} f(x) =$

$f(-3) =$

$\lim\limits_{x \to -3} f(x) =$

Limit: (*geeky math definition for Mr. Kelly*)
Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim\limits_{x \to c} f(x) = R$.

What is a *one-sided* limit?

A **one-sided limit** is the _____ a function approaches as you approach a given _____ from either the _____ or _____ side.

Example 2
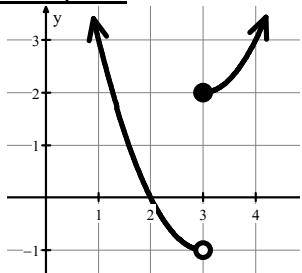
"The limit of f as x approaches 3 from the left side is -1."

$\lim\limits_{x \to 3^-} f(x) = -1$

"The limit of f as x approaches 3 from the right side is 2."

$\lim\limits_{x \to 3^+} f(x) = 2$

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Example 3

| a. $\lim\limits_{x \to -2^-} f(x) =$	b. $\lim\limits_{x \to -2^+} f(x) =$	c. $\lim\limits_{x \to -2} f(x) =$
d. $\lim\limits_{x \to 1} f(x) =$	e. $\lim\limits_{x \to 0} f(x) =$	f. $\lim\limits_{x \to 3^-} f(x) =$
g. $\lim\limits_{x \to -1} f(x) =$	h. $\lim\limits_{x \to -3} f(x) =$	i. $f(-2) =$

j. $f(1) =$
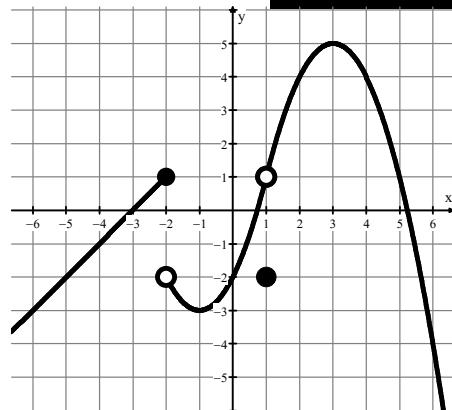
When does a limit not exist?

1.

2.

3.

Example 4

Sketch a graph of a function g that satisfies all of the following conditions.

  a. $g(3) = -1$
  b. $\lim\limits_{x \to 3} g(x) = 4$
  c. $\lim\limits_{x \to -2^+} g(x) = 1$
  d. g is increasing on $-2 < x < 3$
  e. $\lim\limits_{x \to -2^-} g(x) > \lim\limits_{x \to -2^+} g(x)$
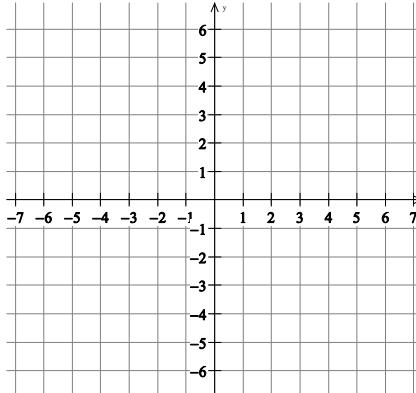
Example 5

Write **T** (true) or **F** (false) under each statement. Use the graph on the right.

a. $\lim\limits_{x \to -1^+} f(x) = 1$	b. $\lim\limits_{x \to 2} f(x) = 2$	c. $\lim\limits_{x \to 1^-} f(x) = 1$
d. $\lim\limits_{x \to 1^+} f(x) = 2$	e. $\lim\limits_{x \to 1} f(x) =$ does not exist	
f. $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x)$	g. $\lim\limits_{x \to 2} f(x) =$ does not exist	
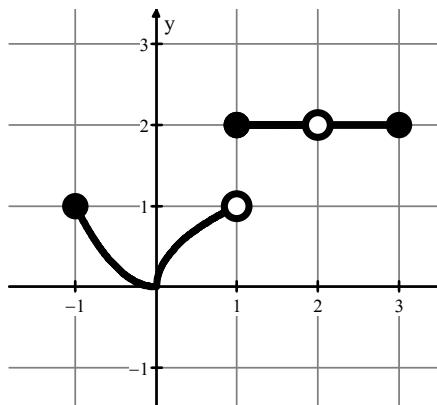
1.1 Limits Graphically

Calculus   Name: _____

For 1-5, give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."

1.

a. $\lim\limits_{x\to-1^-} f(x) =$

b. $f(1) =$

c. $\lim\limits_{x\to0} f(x) =$

d. $\lim\limits_{x\to2^+} f(x) =$

e. $f(-1) =$

f. $f(2) =$

g. $\lim\limits_{x\to-1^+} f(x) =$

h. $\lim\limits_{x\to1^-} f(x) =$

i. $\lim\limits_{x\to2} f(x) =$
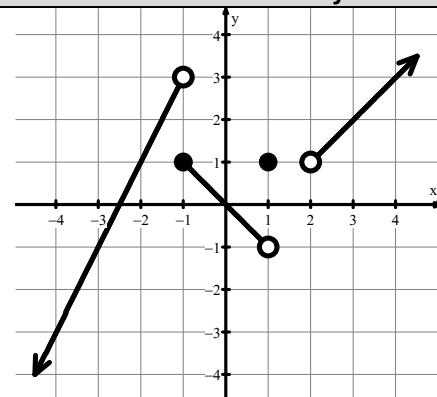
2.

a. $\lim\limits_{x\to-3} f(x) =$

b. $f(1) =$

c. $\lim\limits_{x\to1} f(x) =$

d. $\lim\limits_{x\to-2^+} f(x) =$

e. $f(3) =$

f. $\lim\limits_{x\to-2^-} f(x) =$

g. $\lim\limits_{x\to-2} f(x) =$

h. $f(-2) =$

i. $f(4) =$
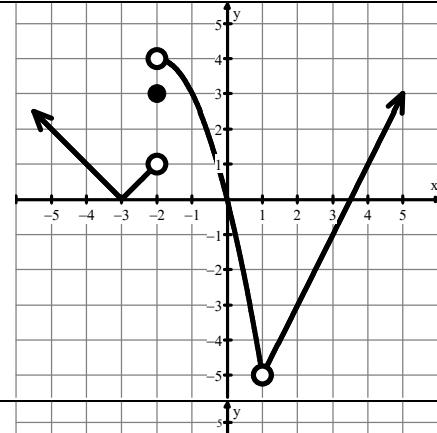
3.

a. $\lim\limits_{x\to3^+} f(x) =$

b. $f(3) =$

c. $\lim\limits_{x\to0} f(x) =$

d. $\lim\limits_{x\to3} f(x) =$

e. $f(0) =$

f. $\lim\limits_{x\to3^-} f(x) =$

g. $\lim\limits_{x\to0^+} f(x) =$

h. $f(1) =$

i. $f(-1.6) =$
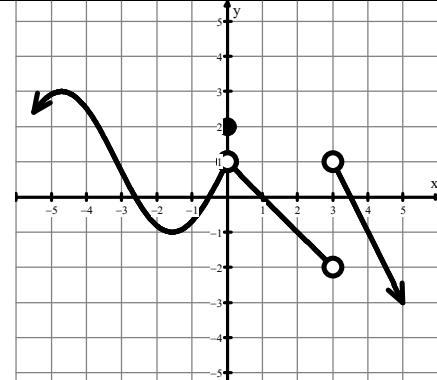
4.

a. $\lim\limits_{x\to-1^-} f(x) =$

b. $f(2) =$

c. $\lim\limits_{x\to2} f(x) =$

d. $\lim\limits_{x\to-1} f(x) =$

e. $f(4) =$

f. $\lim\limits_{x\to1^-} f(x) =$

g. $\lim\limits_{x\to-1^+} f(x) =$

h. $f(1) =$

i. $\lim\limits_{x\to4} f(x) =$
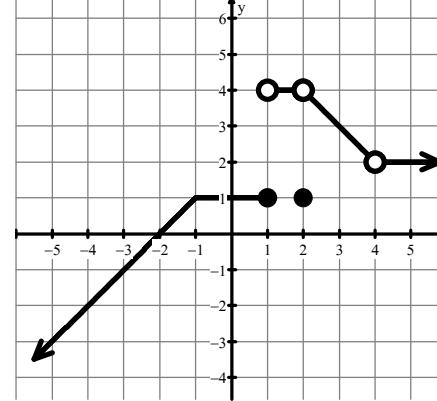
5.

a. $\lim\limits_{x\to 3^-} f(x) =$

b. $f(-1) =$

c. $\lim\limits_{x\to -3} f(x) =$

d. $\lim\limits_{x\to -1} f(x) =$

e. $f(-3) =$

f. $\lim\limits_{x\to 3^+} f(x) =$

g. $f(3) =$

h. $\lim\limits_{x\to 0} f(x) =$

i. $f(-4) =$
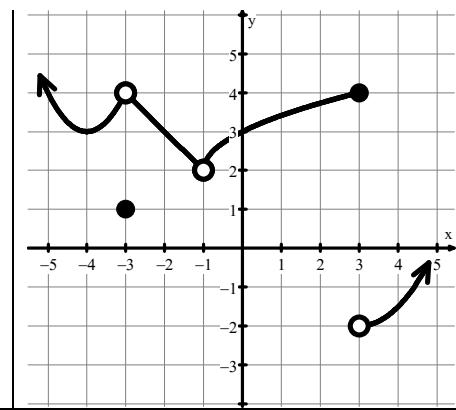

6. Sketch a graph of a function f that satisfies all of the
   following conditions.

   a. $f(-2) = 5$

   b. $\lim\limits_{x\to -2} f(x) = 1$

   c. $\lim\limits_{x\to 4^+} f(x) = 3$

   d. f is increasing on $x < -2$

   e. $\lim\limits_{x\to 4^-} f(x) < \lim\limits_{x\to 4^+} f(x)$
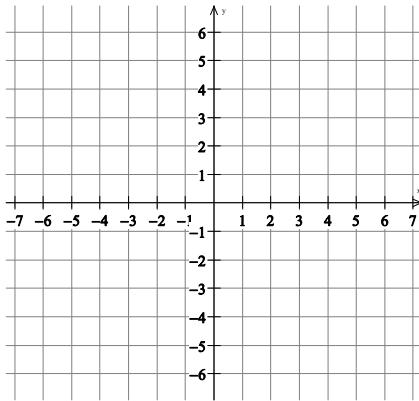
7. Sketch a graph of a function g that satisfies all of the
   following conditions.

   a. $g(1) = 3$

   b. $\lim\limits_{x\to 1} g(x) = -2$

   c. $\lim\limits_{x\to -3^+} g(x) = 5$

   d. g is increasing only on $-5 < x < -3$ and $x > 1$

   e. $\lim\limits_{x\to -3^-} g(x) > \lim\limits_{x\to -3^+} g(x)$
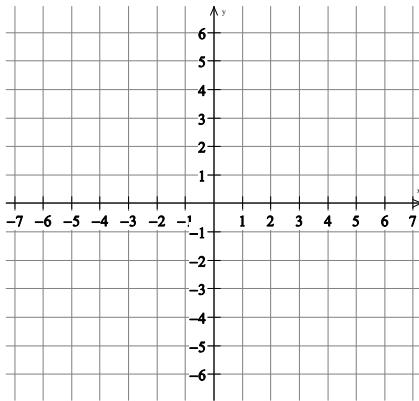
8. Sketch a graph of a function h that satisfies all of the
   following conditions.

   a. $\lim\limits_{x\to 3} h(x) = h(-2) = 1$

   b. $h(3)$ is undefined.

   c. $\lim\limits_{x\to -2^-} h(x) < \lim\limits_{x\to -2^+} h(x)$

   d. h is constant on $-2 < x < 3$ and decreasing
      everywhere else.
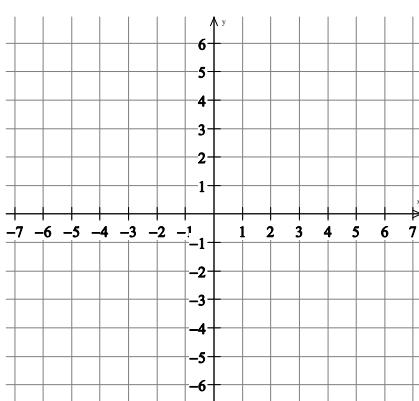
1.1 Limits Graphically

1. The graph of the function f is shown. Which of the following statements about f is true?
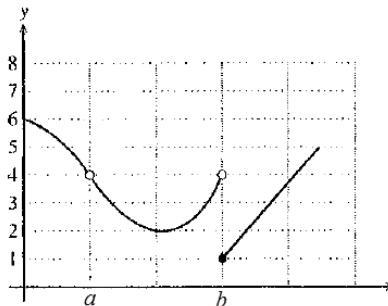
(A) $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to b} f(x)$

(B) $\lim\limits_{x \to a} f(x) = 4$

(C) $\lim\limits_{x \to b} f(x) = 4$

(D) $\lim\limits_{x \to b} f(x) = 1$

(E) $\lim\limits_{x \to a} f(x)$ does not exist.

2. The figure below shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim\limits_{x \to 2^-} f(x)$ exists.

II. $\lim\limits_{x \to 2^+} f(x)$ exists.

III. $\lim\limits_{x \to 2} f(x)$ exists.

Graph of f
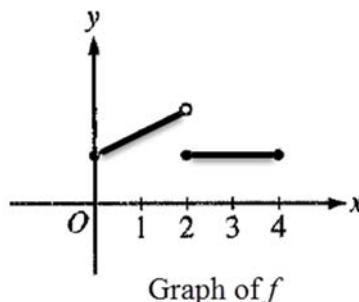
(A) I only   (B) II only   (C) I and II only   (D) I and III only   (E) I, II, and III

3. If $[x]$ represents the greatest integer that is less than or equal to x, then $\lim\limits_{x \to 0^-} \dfrac{2}{[x]} =$

(A) -2 (B) -1 (C) 0 (D) 2 (E) the limit does not exist

4. Consider the function $y = f(x)$ shown below. Which of the following statements is true?
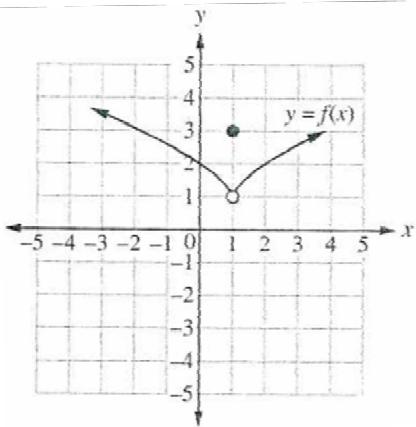
(A) $\lim\limits_{x \to 1} f(x) = 3$

(B) $f(1) = 1$

(C) $f(x)$ is continuous for all x.

(D) $\lim\limits_{x \to 1} f(x) = f(1)$

(E) None of the above