Pre-Calculus Midterm Review Name_____

1) Kim's college uses a 5.0 grade scale as follows: A = 5.0, B+ = 4.5, B = 4.0, C+ = 3.5, C = 3.0, D+ = 2.5, D = 2, F = 1.0. It then weights the grade by the number of semester hours for each course. Compute Kim's grade point average based on the report at the right.

Course	Grade	Hours
Biology	4.5 B+	4
Bio Lab	5.0 A	2
Calculus	4.0 B	5
Am. History	3.5 C+	4
French	3.0 C	4

19 total

$4(4.5)+2(5.0)+5(4.0)+4(3.5)+4(3.0) = \frac{74}{19} =$ **3.89 GPA**

2) An astronaut standing on the rim of a crater on the moon tosses a ball upward at a velocity of 10 m/sec. The rim of the crater is 200 meters above the floor.

v_0 h_0

a) Write an equation for the height h (in meters above the crater floor) of the ball after t seconds. Use the formula $h = -\frac{1}{2}gt^2 + v_0t + h_0$, where $g = 1.625\,m/sec^2$.

$h = -\frac{1}{2}(1.625)t^2 + 10t + 200$

$h(t) = -0.8125t^2 + 10t + 200$

b) Predict the height of the ball above the crater floor after 5 seconds.

$h(5) = -0.8125(5)^2 + 10(5) + 200$

$= 229.6875$ m

c) At what time will the ball hit the crater floor?

$a = -0.8125 \quad b = 10 \quad c = 200$

$0 = -0.8125t^2 + 10t + 200$

$X = \frac{-(10) \pm \sqrt{(10)^2 - 4(-0.8125)(200)}}{2(-0.8125)}$

$= \frac{-10 \pm \sqrt{750}}{-1.625}$

$\frac{-10 + \sqrt{750}}{-1.625} = -10.699$ (time can't be negative)

$\frac{-10 - \sqrt{750}}{-1.625} =$ **23.01 sec**

3) An isotope of strontium, ^{89}Sr, is used in treatment of bone cancer. It has a half-life of 25.4 days. To the nearest hundredth gram, how much of a 1-gram sample will remain after 60 days?

$y = a \cdot b^x$

$\frac{0.5}{1} = \frac{1 \cdot b^{25.4}}{1}$

$(0.5)^{\frac{1}{25.4}} = (b^{25.4})^{1/25.4}$

$b = 0.97$

$y = 1(0.97)^{60}$

$y = 0.16$ grams

4) Suppose the scale change $S(x,y)=\left(2x,\frac{y}{3}\right)$ is applied to the graph of $y=\frac{1}{|x|}$. Write an equation for the image.

$3y = \frac{1}{\left|\frac{x}{2}\right|}$

5) Suppose $T:(x,y)\to(x-6,y+9)$ is applied to the graph of $y=|x|$. Write an equation for the image.

$y-9 = |x+6|$

-OR-

$y = |x+6| + 9$

6) Tell whether the function f with equation $f(x)=2x^3+4$ is even, odd, or neither. Support your answer algebraically.

$f(-x) = f(x) \Rightarrow$ EVEN

$f(-x) = -f(x) \Rightarrow$ ODD

$-f(x) = -(2x^3+4) = -2x^3-4$

$f(-x) = 2(-x)^3 + 4$

$= -2x^3 + 4 \quad \neq f(x)$

$\neq -f(x)$

Neither

In 7 and 8, let $f(x)=4x-3$ and $g(x)=\frac{5}{x}$.

7) Write an expression for $g(f(x))$.

$g(4x-3)$

$= \frac{5}{4x-3}$

8) Give the domain of $g\circ f$.

$f(x) = 4x-3 \to$ all $\mathbb{R}$

$g(f(x))$

$4x-3 \neq 0$

$+3 \quad +3$

$\frac{4x}{4} \neq \frac{3}{4}$ $\quad x \neq 3/4$

D: $\{x \mid x \in \mathbb{R},\ x \neq 3/4\}$

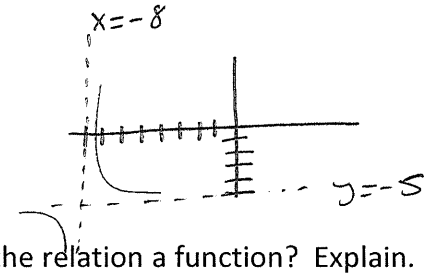
9) Expand $(x+y)^7$

$_7C_0x^7y^0 + {}_7C_1x^6y^1 + {}_7C_2x^5y^2 + {}_7C_3x^4y^3 + {}_7C_4x^3y^4 + {}_7C_5x^2y^5 + {}_7C_6x^1y^6 + {}_7C_7x^0y^7$

$1x^7 + 7x^6y^1 + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7x^1y^6 + 1y^7$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

10) Consider the equation $f(x)=\frac{7}{x+8}-5$. ↓5 ← 8



a) Give the equations for the asymptotes of the graph.

$x=-8$

$y=-5$

b) Is the relation a function? Explain.

yes, passes VLT.

c) State the domain.

$\{x \mid x \neq -8\}$

d) State the range.

$\{y \mid y \neq -5\}$

e) Find the inverse of the relation.

$x=\frac{7}{y+8}-5$ $\quad +5 \quad +5$

$(y+8)(x+5)=\left(\frac{7}{y+8}\right)y+8$

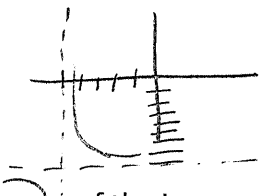
$\frac{(y+8)(x+5)}{x+5}=\frac{7}{x+5}$

$y+8=\frac{7}{x+5}$ $\quad -8 \quad -8$

$\boxed{y=\frac{7}{x+5}-8}$

f) Is the inverse a function? Explain.

yes, passes VLT.



g) State the domain of the inverse.

$\{x \mid x \neq -5\}$

i) State the range of the inverse.

$\{y \mid y \neq -8\}$

11) The chance of having to stop for a train at any railroad crossing is 0.457. What is the probability of having to stop for a train at exactly 4 of the next 7 railroad crossings?

$1-0.457=0.543$

$_7C_4\,(0.457)^4(0.543)^3 = \boxed{0.244}$

$= \boxed{24.4\%}$

12) A test includes 8 true/false questions, 10 multiple choice questions, each with 3 choices, and 5 multiple choice questions, each with 4 choices. What is the probability of guessing correctly on all questions?

$\frac{1}{2^8 \cdot 3^{10} \cdot 4^5} = \boxed{6.46 \times 10^{-11}}$

$= \boxed{6.46 \times 10^{-9}\,\%}$

In 13 and 14, consider the following situation. A manufacturer of high-pressure air bags hired an independent laboratory to determine the internal pressure at which air bags explode. Technicians at the laboratory randomly selected 42 of the air bags, inflated them, and measured the pressure in pounds per square inch (psi) at which each bag burst. They found the following statistical measures for their set of data.

Mean: 223 psi **Variance: 16 psi^2**

Minimum: 208 psi **Maximum: 248 psi**

13) To account for atmospheric pressure, the technicians adjusted the pressure at which the air bag burst by subtracting 125 psi from it. Find the statistical measure for the set of adjusted pressures.

a. mean $223 - 125 = 98$ psi b. variance 16 psi^2 c. range $248 - 208 = 40$ psi

↳ measure of center changes ↑ measures of spread stay the same ↗

14) The company's European division wanted to see the statistics presented in terms of bars rather than psi. One bar is approximately 14.5 psi. Find each statistical measure for the set of original pressures.

a. mean $\frac{223}{14.5} = 15.38$ bar b. variance $\frac{16}{14.5^2} = 0.08$ bar c. range $\frac{40}{14.5} = 2.76$ bar

All divided by scale factor, except variance, divide by (scale factor)^2

15) A code consists of 5 shapes (circle, square, triangle, rectangle, pentagon or oval) followed by 3 letters.

a) How many codes can be made if repeats are allowed?

$6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 26 \cdot 26 \cdot 26 = \boxed{136,670,976}$

b) How many codes can be made if repeats are NOT allowed?

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 26 \cdot 25 \cdot 24 = \boxed{11,232,000}$

16) How many ways can 3 students from a class of 26 be chosen to be given a cookie?

$_{26}C_3 = \boxed{2600}$

17) How many ways can ~~3 teachers be chosen for~~ 3 different prizes (a $20 gas card, a free lunch, a day off) be awarded to 30 teachers?

$_{30}P_3 = \boxed{24,360}$

65 total

18) A box contains 20 pens, 35 pencils and 10 markers. If 10 items are randomly taken from the bag, what is the probability of selecting exactly 3 pens, 6 pencils and 1 marker?

$$\frac{{}_{20}C_3 \cdot {}_{35}C_6 \cdot {}_{10}C_1}{{}_{65}C_{10}} = 0.103 = 10.3\%$$

19) A pair of 5-sided dice are tossed. Let A = {the difference is 1} and B = {2 appears on either die}. $5 \cdot 5 = 25$

a) Find $P(B \mid A)$

$$= \frac{P(B \cap A)}{P(A)}$$

$$= \frac{\frac{4}{25}}{\frac{8}{25}} = \frac{4}{8} \text{ or } \frac{1}{2}$$

b) Find $P(A \mid B)$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{4}{25}}{\frac{9}{25}} = \frac{4}{9}$$

A	B
★(1,2)	★(2,1)
★(2,3)	(2,2)
(3,4)	★(2,3)
(4,5)	(2,4)
(5,4)	(2,5)
(4,3)	(5,2)
★(3,2)	(4,2)
★(2,1)	★(3,2)
8	★(1,2)
	9

20) Walmart customers were randomly questioned. They were asked their age, and whether or not they had minor children. The results are shown in the table below.

	20s	30s	40s	50s	60s	
Have minor children	10	15	25	8	2	60
Do not have minor children	5	7	3	15	25	55
	15	22	28	23	27	115

a) What percent of people in their 40s do not have minor children?

$$\frac{3}{28} = 0.107 = 10.7\%$$

b) What percent of all customers surveyed have minor children?

$$\frac{60}{115} = 0.522 = 52.2\%$$

For questions 21-23, use the teacher salaries by years of experience shown below.

Years of Experience	0	5	10	15	20
Teacher Salary	$35,000	$43,000	$50,000	$59,000	$63,000

21) a. Find an equation for the line of best fit for the data. Round to the nearest thousandth.

$$y = 1440x + 35600$$

b. Find the sum of the squared residuals. Round to the nearest thousandth.

$$\Sigma x^2 = 56{,}00{,}000$$

22) a. Find an exponential equation for the data above. Round to the nearest thousandth.

$y = 36287.254(1.030)^x$

b. Find the sum of the squared residuals. Round to the nearest thousandth.

$\Sigma x^2 = 17{,}034{,}008.8$

23) Using information from 21 & 21, which model (linear or exponential) best fits the data provided. Explain.

Linear is better as it has a smaller sum of squared residuals.

For questions 24-26, use the following two sets of data. The ages of people attending concerts is given below:

Shawn Mendes: 11, 16, 12, 13, 15, 17, 18, 23, 41, 13

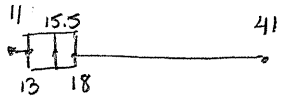
Willy Nelson: 65, 53, 24, 18, 22, 80, 44, 38, 61

24) Find the mean, standard deviation, and 5-number summary for each concert.

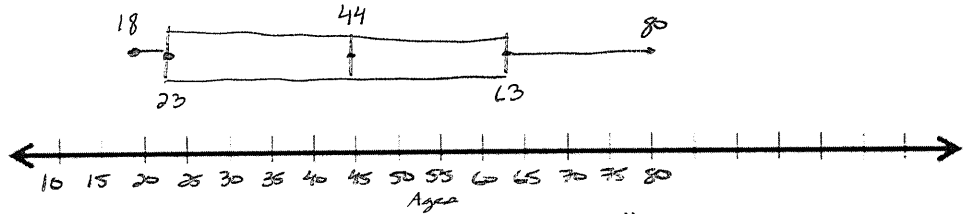
	Shawn Mendes	Willy Nelson
a. Mean	$\bar{x}$: 17.9	$\bar{x}$: 45
b. Standard Deviation	S_x: 8.84	S_x: 21.49
c. Minimum	11	18
d. Quartile 1	13	23
c. Median	15.5	44
d. Quartile 3	18	63
e. Maximum	41	80

25) Create a box plot for each concert.

Shawn Mendes



Willy Nelson



26) Compare and contrast the two data sets. (Shape, Center, Spread)

- Both skewed right

Mendes
- mean 17.9, median 15.5 (younger fans)
- range 30
- smaller S.D. → ages of fans clustered closer together

Nelson
- mean 45 & median 44 (older fans)
- range 62
- wide range of age of fans