

FST Midterm Review

Name \_\_\_\_\_

For questions 1-3 use the following two sets of data.

**Execco** Salaries (\$ 000) for eight employees.

**Flatco** Salaries (\$ 000) for eight employees.

98, 69, 48, 34, 24, 24, 24, 20

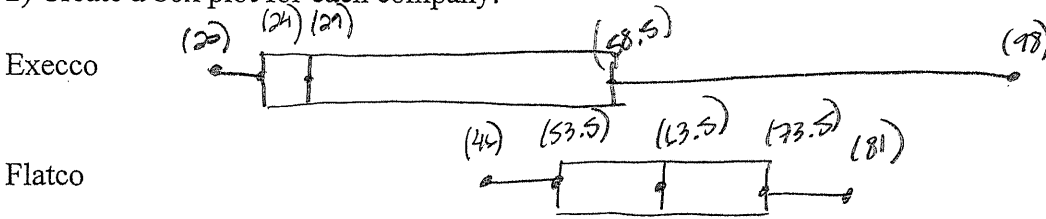
81, 76, 71, 66, 61, 56, 51, 46

1) Find the mean and standard deviation for each company.

a) Mean: Execco  $\$ 42,625$  (42.625) Flatco  $\$ 63,500$  (63.5)

b Standard Deviation: Execco  $\$ 27,821$  (27.821) Flatco  $\$ 12,247$  (12.247)

2) Create a box plot for each company.



3) Write a few sentences to compare and contrast of the two data sets. (Shape, Center, Spread) Execco data is skewed right. Flatco is symmetric. Execco's average salary is around \$20,000 below Flatco's. The salary range for Execco is \$78,000. The range for Flatco is \$35,000.

4) Use Susan's report card at the right to find her GPA.

Susan's Report Card

Course	Credit	Letter	Grade Points
Keyboarding	1	A+	4.00
AP Calculus	5	B-	2.67
Honors English	4	B+	3.33
Regular Level Art History	3	D-	0.67
Regular Level Statistics	3	C	2.00

$$4.00(1) + 2.67(5) + 3.33(4) + 0.67(3) + 2.00(3) = 38.68$$

total grade points

$$\frac{38.68 \text{ grade points}}{1 + 5 + 4 + 3 + 3 \text{ credits}} = \boxed{2.42 \text{ GPA}}$$

5) Plutonium-239 is radioactive decay used in nuclear weapons and submarines. It has a half-life of 24,110 years. How much of a 200 pounds sample will remain after 9238 years?

$$y = a \cdot b^x$$

$$0.5 = 1 \cdot b^{24,110}$$

$$0.5^{\frac{1}{24,110}} = (b^{24,110})^{\frac{1}{24,110}}$$

$$\boxed{b = 0.99997}$$

$$y = 200(0.99997)^{9238}$$

$$\boxed{y = 151.6 \text{ lbs}}$$

#6 in-class

#6 in-class

#2 in-class

6) A rock is thrown upward from the edge of an 80 m cliff overlooking Lake Superior, with an initial velocity of 17 m/s.

$$h = -\frac{1}{2}gt^2 + v_0t + h_0, \text{ where } g = 9.8\text{m/sec}^2$$

a) Write an equation that models the height  $h$  of the rock in terms of time  $t$ .

$$h = -\frac{1}{2}(9.8)t^2 + 17t + 80$$

$$h = -4.9t^2 + 17t + 80$$

b) Predict the height of the rock at 4 seconds.

$$h = -4.9(4)^2 + 17(4) + 80 = 69.6\text{m}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

c) At what time will the rock hit the ground?  $a = -4.9$   $b = 17$   $c = 80$

$$0 = -4.9t^2 + 17t + 80 \quad \rightarrow \quad -17 \pm \sqrt{1857} \quad \rightarrow \quad \frac{-17 + \sqrt{1857}}{-9.8} = -2.66 = x$$

$$x = \frac{-17 \pm \sqrt{(17)^2 - 4(-4.9)(80)}}{2(-4.9)} = \frac{-17 \pm \sqrt{1857}}{-9.8} \quad \rightarrow \quad \frac{-17 - \sqrt{1857}}{-9.8} = 6.13 = x$$

Use the table for Questions 7-10. The table shows the relationship between the times a student spends typing each week and his percent improvement on speed assessments.

Practice hours	6	8	10	12	14	16	18
Percent improvement	25	26	29	36	30	35	39

Round answers to the thousandths (3 decimal places)

7) a) Find an equation for the line of best fit for the data.

STAT  $\rightarrow$  CALC #4 LinReg

$$y = 1.089x + 18.357$$

b) Find the sum of the squared residuals.

① STAT #1: Edit Go up to L3 and Stat #7: Resid ENTER

② STAT  $\rightarrow$  CALC 1: 1-Var Stats L3 ( $\Sigma x^2$ )

$$\Sigma x^2 = 36.821$$

8) a) Find an exponential equation for the data above.

STAT  $\rightarrow$  CALC #0 ExpReg

$$y = 20.387(1.036)^x$$

b) Find the sum of the squared residuals.

Repeat 7b) directions

$$\Sigma x^2 = 37.423$$

9) a) Find a quadratic equation for the data above.

STAT  $\rightarrow$  CALC #5 QuadReg

$$y = -0.003x^2 + 1.161x + 17.976$$

b) Find the sum of the squared residuals.

Repeat 7b) directions

$$\Sigma x^2 = 36.810$$

10) Which model (linear, exponential, or quadratic) best fits the data above. Explain.

Quadratic because it has the smallest sum of the squared residuals.

odd  
 $f(-x) = -f(x)$

even  
 $f(-x) = f(x)$

11) Is  $f(x) = 5x^3 - x$  even, odd or neither? Prove it algebraically.

$$f(-x) = 5(-x)^3 - (-x)$$

$$-f(x) = -5x^3 + x$$

$$= 5(-x^3) + x$$

$$= -5x^3 + x \neq 5x^3 - x \rightarrow \boxed{\text{NOT even}}$$

$$= -f(x) \rightarrow \boxed{\text{ODD}}$$

12) A set of data is translated. Find the missing values in the table.

	Original Data	Transformed Data
cases	10	? $\boxed{10}$
mean	? $\boxed{63.5}$	53
standard deviation	8.03	? $\boxed{8.03}$
median	70	$\boxed{-10.5} = 59.5$
range	23	? $\boxed{23}$
IQR	12	12

• Measures of Center Change (mean, median, mode)

• Measures of Spread stay the same (range, IQR, SD, variance)

13) Let  $f(x) = \frac{6}{x-2}$  and  $g(x) = 5x$ .

a) Write an expression for  $f(g(x))$ .

$$\boxed{\frac{6}{5x-2} = f(g(x))}$$

b) Give the domain of  $f \circ g$ .

Domain of  $g(x) \rightarrow \text{all } \mathbb{R}$

Domain of  $f(g(x)) \rightarrow 5x-2 \neq 0$   
 $+2 \quad +2$   
 $5x \neq 2$   
 $\frac{5x}{5} \neq \frac{2}{5}$   
 $x \neq \frac{2}{5}$

$$\boxed{D: \{x \mid x \in \mathbb{R}, x \neq \frac{2}{5}\}}$$

14) A certain hyperbola H is a translation image of the graph of  $y = \frac{1}{x}$  and has asymptotes  $x = 2$  and  $y = -5$ .

a) Give an equation for H.

$$\boxed{y = \frac{1}{x-2} - 5}$$

b) State the domain and range of the image H.

$$D: \{x \mid x \in \mathbb{R}, x \neq 2\} \quad R: \{y \mid y \in \mathbb{R}, y \neq -5\}$$

15) Using the parent function  $y = \frac{1}{x}$ , write the equation for its image under the

following transformations.  $S(x, y) \rightarrow \left(3x, \frac{y}{5}\right)$ .

$$\boxed{Sy = \frac{1}{\frac{x}{3}}}$$

OR  

$$\boxed{Sy = \frac{3}{x}}$$

$$\frac{Sy}{5} = \frac{\frac{3}{x} \cdot \frac{1}{5}}{\frac{1}{5}}$$

$$\boxed{y = \frac{3}{5x}}$$

16) Use the relation to answer the questions below:  $y = \frac{1}{x+5} - 3$

a) Find the inverse of the relation.

b) Is the inverse a function?

$$x = \frac{1}{y+5} - 3$$

$$+3 \quad +3$$

$$y+5 = \frac{1}{x+3}$$

$$-5 \quad -5$$

yes, passes V.I.T.

$$(y+5)(x+3) = \left(\frac{1}{y+5}\right)(y+5)$$

$$y = \frac{1}{x+3} - 5$$

$$\frac{(y+5)(x+3)}{x+3} = \frac{1}{x+3}$$

c) State the domain of the inverse.

$$D: \{x \mid x \in \mathbb{R}, x \neq -3\}$$

d) State the range of inverse.

$$R: \{y \mid y \in \mathbb{R}, y \neq -5\}$$

17) Find the number of ways in which a committee of 2 people from a group of 16 people can be selected.

$$16^C_2 = \boxed{120}$$

18) First-through fourth-place prizes are to be awarded in an essay contest. In how many ways can the winners be selected from among 125 entries?

$$125^P_4 = \boxed{232,593,600}$$

19) A test includes 12 multiple-choice questions each with 5 choices and 8 true/false questions. What is the probability that you guess correctly on all of the questions?

$$\frac{1}{5^{12} \cdot 2^8} = \frac{1}{6.25 \times 10^{10}} = \boxed{1.6 \times 10^{-11} \text{ or } 1.6 \times 10^{-9} \%}$$

20) In a recent survey of 30 students, 25 students favored an earlier opening time for the school cafeteria and 5 opposed it. Find the probability that in a random sample of 8 respondents from this survey, exactly 6 favored the earlier opening time and exactly 2 opposed it.

$$\frac{25^C_6 \cdot 5^C_2}{30^C_8} = \frac{1,771,000}{5,852,725} = \boxed{0.303 = 30.3\%}$$

21) The choices for a sandwich are 4 different meats and 5 different cheeses. How many different sandwiches of 2 meats and 2 cheeses could you make?

$$4^C_2 \cdot 5^C_2 = \boxed{60}$$

22) Suppose an 8-character ID number consists of a letter, A-Z, followed by 5 digits from 0 to 9, followed by two more letters, A-Z.

a) How many ID numbers are possible if repeats are allowed?

$$\underline{26} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{26} \cdot \underline{26} = \boxed{1,757,600,000}$$

b) How many ID numbers are possible if no repeats are allowed?

$$\underline{26} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{25} \cdot \underline{24} = \boxed{477,440,000}$$

23)

A group of students was examined to investigate patterns of enrollment in foreign language classes.

	Freshman	Sophomore	Junior	Senior	
Spanish	8	11	5	2	26
French	2	5	0	1	8
No language	5	2	10	13	30
	15	24	15	16	70

a) What percent of sophomores took Spanish?

$$\frac{11}{24} = \boxed{45.8\%}$$

b) What percent of all students took no language?

$$\frac{30}{70} = \boxed{42.9\%}$$

24) During the 2008-09 NBA regular season Ray Allen of the Boston Celtics had a free throw percentage of 0.952. If he were to shoot 10 consecutive free throws, what is the probability that he would make 8 of them?

$${}_{10}C_8 (0.952)^8 (0.048)^2 = \boxed{0.06995 = 6.995\%}$$

25) A pair of fair 6-sided dice is tossed. Let A = {sum of 6} and B = {5 on 1st die}.

a) Find  $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\frac{\frac{1}{36} \cdot \frac{36}{6}}{\frac{6}{36}} = \boxed{\frac{1}{6}}$$

b)  $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$$\frac{\frac{1}{36} \cdot \frac{36}{5}}{\frac{5}{36}} = \boxed{\frac{1}{5}}$$

A	B
1,5	5,1
2,4	5,2
3,3	5,3
4,2	5,4
5,1	5,5
	5,6

25) Expand  $(x+y)^8$ .

$$= {}_8C_0 x^8 y^0 + {}_8C_1 x^7 y^1 + {}_8C_2 x^6 y^2 + {}_8C_3 x^5 y^3 + {}_8C_4 x^4 y^4 + {}_8C_5 x^3 y^5 + {}_8C_6 x^2 y^6 + {}_8C_7 x^1 y^7 + {}_8C_8 x^0 y^8$$

$$= 1x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8x^1y^7 + 1y^8$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1