FST 10.1 Notes
Topic: Combinations
GOAL
Exhibit situations where combinations are desired. Derive a formula for the number of combinations of n things r at a time.

F Use combinations to compute the number of ways of selecting objects.

G Compute probabilities involving combinations in non-binomial problems.

Vocabulary

combination

number of combinations of *n* things taken *r* at a time, ${}_nC_r \left(\binom{n}{r}\right)$

Warm up
In the 26-letter English alphabet, how many different 2-letter initials are possible:

a) If the same letter can be repeated? $26 \cdot 26 = 676$

b) If the same letter cannot be repeated? $26 \cdot 25 = 650$

Permutations = is the arrangement of n things taken r at a time. ★ Order matters

Theorem (Alternate Formula for ${}_nP_r$)

$${}_nP_r = \frac{n!}{(n-r)!}$$

Combination = is the number of n things taken r at a time. ★ Order does not matter.

${}_nC_r$ or $\binom{n}{r}$

Theorem (Formula for ${}_nC_r$)

For all integers n and r, with $0 \le r \le n$, ${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{(n-r)!r!}$.

Evaluate.

${}_{10}C_6$ $\quad \frac{{}_nP_r}{r!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{5040}{24} = \boxed{210}$

$\binom{17}{3} = {}_{17}C_3 = \frac{{}_{17}P_3}{3!} = \frac{17 \cdot 16 \cdot 15}{3 \cdot 2 \cdot 1} = \frac{4080}{6} = \boxed{680}$

Example 1 P C
Determine whether the problem involves permutations or combinations.

a) How many ways are there to choose a committee of 2 people from a group of 7 people? C $7C_2 = 21$

b) How many ways are there to choose a chairperson and a co-chairperson from a group of 7 people? P $7P_2 = 42$

c) How many ways are there to award first prize, second prize, and third prize to a group of 8 entrants in a contest? P $8P_3 = 336$

d) How many ways are there to give 3 honorable mention awards to a group of 8 entrants in a contest? C $8C_3 = 56$

Example 2
A class has 10 boys and 15 girls. A committee of 5 is to be chosen to represent the class at a group meeting, so it is decided to pick 2 boys and 3 girls. How many different committees are possible? **If 5 students are chosen from the class, what is the probability that exactly 2 of them are boys and 3 are girls?**

$10C_2 \cdot 15C_3 = 20,475$ committees

$\frac{20,475}{25C_5} = \frac{20,475}{53,130} = 0.385 = 38.5\%$

Example 3
Bill drives to school and the route he takes has 5 stoplights. If he reaches all the lights on green, then it takes him 12 minutes to get to school. But every stoplight he reaches on red slows him down about a minute. In how many ways can he be slowed down to get to school in about 15 minutes? $15 - 12 = 3$ extra minutes

$5C_3 = 10$ ways

Example 4
To win a particular lottery game, it is necessary to match the numbers on 5 balls that are randomly picked from 50 balls numbered 1 to 50 and the number on a special black ball that is picked from 9 other balls numbered 1 to 9. What is the probability that a single lottery ticket will match the 5 balls and the special black ball?

$50C_5 \cdot 9C_1 = 19,068,840$

$\frac{1}{19,068,840} = 5.24 \times 10^{-8}$

$5.24 \times 10^{-6}\%$