FST 10.5 Notes
Topic: Binomial Probabilities
GOAL
This lesson shows why ${}_nC_k p^k q^{1-k}$ works and applies it to a variety of situations.

SPUR Objectives

H Determine probabilities in situations involving binomial experiments.

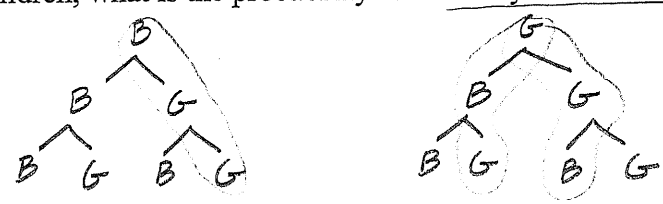
Vocabulary

binomial experiment

- Fixed # of repeated trials
- Each trial has 2 possible outcomes
- P(success) is same in each trial
- Trials are independent

binomial experiment An experiment with a fixed number of independent trials, each with only two possible outcomes, often called success and failure, and each with the same probability of success.

Warm up
A family has 3 children, what is the probability that exactly one child is a boy?



$2 \cdot 2 \cdot 2 = 2^3$
$= 8$

$\frac{3}{8} = 0.375$

A **binomial experiment** has the following features:
1) There are repeated situations, called *trials.*
2) There are a fixed number of trials.
3) For each trial, there are only two possible outcomes, often called *success* (S) and *failure* (F).
4) The probability of success is the same in each trial.
5) The trials are independent events.

Binomial Probability Theorem

Suppose that in a binomial experiment with n trials, the probability of success is p in each trial and the probability of failure is q, where $q = 1 - p$. Then

$$P(\text{exactly } k \text{ successes}) = {}_nC_k \cdot p^k q^{n-k} = \binom{n}{k} p^k q^{n-k}.$$

Binomial Probability Formula (another version of the formula)

$$P = {}_nC_r p^r (1-p)^{n-r}$$

n = # of trials
r = successes
little p = probability of success
1-p = probability of failure

Warm up (Solve using the Binomial Probability Formula)
A family has 3 children, what is the probability that exactly one child is a boy?

$n = 3$
$r = 1$
$p = 0.5$
$1-p = 0.5$

$$P = {}_3C_1 (0.5)^1 (0.5)^2 = \boxed{0.375}$$

Example 1
Some hereditary diseases are inherited by one-forth of the offspring of the families in which the hereditary gene is present. If such a family has four offspring, what is the probability that exactly one of the offspring inherits the gene?

$n = 4$
$r = 1$
$p = 0.25$
$1-p = 0.75$

$$P = {}_4C_1 (0.25)^1 (0.75)^3 = \boxed{0.4219}$$

Example 2
Suppose you feel that you have a 90% probability of correctly answering any question on an upcoming history test. If there are ten questions on the test, what is the probability that you will correctly answer 80% or more of the questions ?

+ 90% + 100%

$n = 10$
$r = 8, 9, 10$
$p = 0.9$
$1-p = 0.1$

$$P = {}_{10}C_8 (0.9)^8 (0.1)^2 + {}_{10}C_9 (0.9)^9 (0.1)^1 + {}_{10}C_{10} (0.9)^{10} (0.1)^0$$

$$= 0.1937 + 0.3874 + 0.3487$$

$$= \boxed{0.9298}$$

10-5 EXIT SLIP

Suppose that the probability of an Iphone being defective is 2%. What is the probability that 4 Iphones are defective in a shipment of 50 Iphones?

$$= {}_{50}C_4 (.02)^4 (1-.02)^{50-4}$$

$${}_{50}C_4 (.02)^4 (.98)^{46} = .0145$$

$$= 1.5\%$$