FST 2-4 Notes
Topic: Exponential Functions
GOAL:
Review the properties of exponential functions.

SPUR Objectives

E Describe properties of exponential functions.

Vocabulary

growth factor WHAT INITIAL AMOUNT IS REPEATEDLY MULTIPLIED BY

exponential function with base *b* $f(x) = ab^x$

exponential growth function $b > 1$

exponential growth curve CURVE GOES UP FROM LEFT TO RIGHT

exponential decay function $0 < b < 1$

asymptote A LINE THAT THE GRAPH APPROACHES BUT NEVER TOUCHES OR CROSSES

Warm up
Suppose you earn $1000 on a job and 10% is taken out for income tax. Then your employer gives you a 10% raise. How much will you have after the raise and tax?

$1000(1 - .10) = \$900$
$900(1 + .10) = \$990$

Definition of Exponential Function

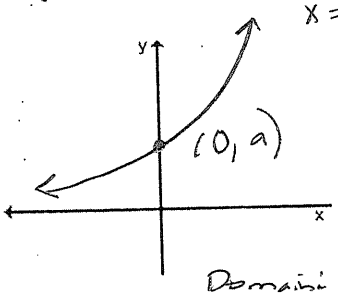
An **exponential function with base *b*** and initial value *a* is a function with an equation of the form

$$f(x) = ab^x,$$

where $a \neq 0$, $b > 0$, and $b \neq 1$.

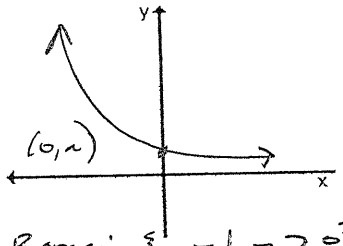
a = INITIAL AMOUNT
x = time (# of growth or decay periods)
$y = ab^x$

Exponential Growth

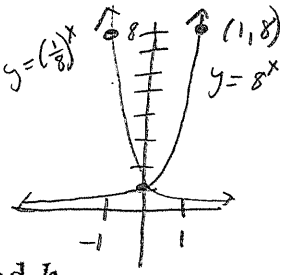


Domain: $\{x \mid x \in \mathbb{R}\}$

Exponential Decay



Range: $\{y \mid y > 0\}$



Compare and contrast the graphs of the three functions f, g, and h, where $f(x) = 8^x$, $g(x) = \left(\frac{1}{8}\right)^x$, and $h(x) = 8^{-x}$ for all values of x.

* LOOK AT TABLE OF CALC

$f(x) = 8^x$ $b = 8$ $(b > 1)$ exponential growth

SAME $\begin{cases} g(x) = (1/8)^x & b = 1/8 \quad (0 < b < 1) \text{ exponential decay} \\ h(x) = 8^{-x} = \frac{1}{8}^x & b = 1/8 \quad (0 < b < 1) \text{ exponential decay} \end{cases}$

Domain: $\{x \mid x \in \mathbb{R}\}$ Range: $\{y \mid y > 0\}$

1 + % as decimal

100%
+ 2.3%
102.3%
1.023

1 − % as decimal

10 − .9%
99.1%
0.991

1. Presently, the towns of Scarcedale and Ampleton both have approximately 8500 residents. Over the next 5 years, the population of Ampleton is expected to increase by approximately 2.3% per year, while the population of Scarcedale is expected to decrease by about 0.9% each year.
   a. Create equations to describe the population of each town as a function of time.
   b. Compare the projected populations after 5 years.

a) Ampleton ↑ 2.3%

$y = 8500(1.023)^x$

Scarcedale ↓ 0.9%

$y = 8500(.991)^x$

$y = \text{STARTING VALUE}\left(\text{GROWTH FACTOR}\right)^{\text{TIME}}$

$y = ab^x$

b) $y = 8500\ (1.023)^5$

$y = 9523.5$

$= 9524$ people

Pop ↑ by 1024 ppl in 5 yrs

$y = 8500(.991)^5$

$y = 8124.3$

$= 8124$ people

Pop ↓ by 376 ppl in 5 yrs

$9524 - 8124 = 1400$

Pop differs by 1400 in 5 yrs.

$y = ab^x$ (end, start, x ← time, b ↑ rate)

2. Jonas received a letter from his credit union stating that a 5-year CD his parents opened for him had matured and he could choose one of the following options:
   (1) Withdraw the full amount of $14,204.10.
   (2) Use the money to open a new 5-year CD with an APY of 2.81%. (1 + 0.0281)
   (3) Have the money deposited into his interest checking account which earns 0.95% per year.

   Assume Jonas's parents deposited $12,000 into the CD 5 years ago.
   a. What is an equation for the balance of the old CD after 5 years?
   b. Use the equation from Part a to find the annual yield of the old CD.
   c. Compare the results of options (2) and (3).

a) $y = ab^x$

$\frac{14,204.10}{12000} = \frac{12,000}{12000}(1+r)^5$

b) $(1.183675)^{1/5} = ((1+r)^5)^{1/5}$

$r = 0.0343 \rightarrow 3.43\%$

c) (2) $y = 14,204.10(1.0281)^5$

$y = 16,315.13$

(3) $y = 14,204.10(1.0095)^5$

$y = 14891.74$

★ CD yields $1423.39 more than checking.

$$y = a \cdot b^x$$

$y = \text{start} \cdot \text{growth factor}^{\wedge \text{time}}$

FST 2-4 Extra Practice

x	y
0	3
1	6

$a = 3$

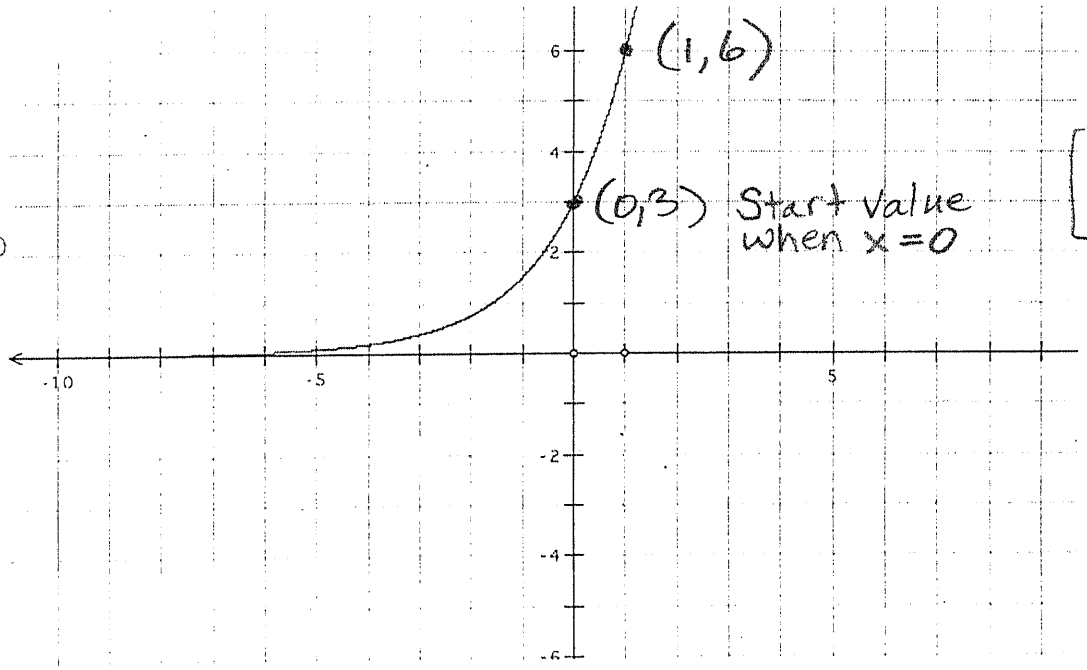
$y = a \cdot b^x$

$y = 3 \cdot b^x$

$6 = 3 \cdot b^1$

$\frac{6}{3} = \frac{3 \cdot b}{3}$

$2 = b$



$y = a \cdot b^x$

$y = 3 \cdot 2^x$

$y = a \cdot b^x$

$y = \text{start} \cdot \text{growth factor}^{\wedge \text{time}}$

FST 2-4 Extra Practice

X	y
0	3
1	6

$a = 3$

$y = 3 \cdot b^x$

$\frac{6}{3} = \frac{3 \cdot b^1}{3}$

$b = 2$

$y = 3(2)^x$