FST 2-5 Notes
Topic: Exponential Models
GOAL
Construct mathematical models of situations that are exponential in nature. Given two points of such a model, find an equation of the form $y = ab^x$ by solving a system of two equations with two unknowns.

SPUR Objectives

F Find and interpret exponential regression and models.

Vocabulary

exponential regression
half-life

1. The population of a certain cell type was observed to be 100 on the second day, and 2700 on the fifth day. Assuming the growth is exponential, find the number of cells present initially, and the number of cells expected on the seventh day.

DAY x	CELLS y	
2	100	> DIVIDE
5	2700	

SUBTRACT

$b^{5-2} = \frac{2700}{100}$

$(b^3)^{1/3} = (27)^{1/3}$

$b = 3$
GROWTH FACTOR

$y = ab^x$

$100 = a(3)^2$

$\frac{100}{9} = \frac{9a}{9}$

$a = \frac{100}{9} \approx 11.\overline{1}$
START VALUE

$y = \frac{100}{9}(3)^x$

$y = \frac{100}{9}(3)^7$

$y = 24,300$ cells on 7th day

$(2, 100)$
$x \quad y$

2. The National Science Foundation (NSF) publishes InfoBriefs, a newsletter containing brief reports highlighting results from recent surveys and analyses. The following data are contained in a January 2009 article about Federal R&D funding.

   a. Use a statistics utility to graph the data and fit an exponential model of the form $f(x) = ab^x$ to the data. Use x as "years after 1990." Report the values of a and b in the exponential model to the nearest thousandth.

	L1 Fiscal year	L2 Federal obligations for research (NSF) ($ millions)
Coded year	0 1990	1690
	1 1991	1785
	2 1992	1868
	3 1993	1882
	4 1994	2040
	5 1995	2149
	6 1996	2188
	7 1997	2249
	8 1998	2289
	9 1999	2506
	10 2000	2726
	11 2001	3044
	12 2002	3260
	13 2003	3609
	14 2004	3771
	15 2005	3743
	16 2006	3791
	17 2007	4051
	18 2008	4358

data already in L1, L2

view scatter plot: 2nd - STAT PLOT
ZOOM - 9:ZoomStat

observe scatter plot - does data appear linear? exponential?

find exponential model: STAT - CALC - 0:ExpReg (enter)

LinearReg
$y = 151.72x + 1423.55$
$r = 0.978$
$\sum x^2 = 608,606.658$

ExpReg
$y = 1625.16(1.056)^x$
$r = .989$ ← closer to 1 so better
$\sum x^2 = 321,953.727$ ← smaller so better

b. Use your graph to describe how well the exponential curve you have modeled fits the points on the scatterplot.

find & graph exponential model: STAT - CALC - 0:ExpReg (enter)
L1,L2,Y1 (VARS - Y-VARS - Function - Y1)

$y = a(b)^x$
$y = 1625.16(1.056)^x$

$r = 0.989$ → Exp. better fit than linear

c. Calculate the residuals for 1998 and 2004.

8 14

find ordered pairs (predicted values) for model: 2nd TABLE

adjust table: 2nd TBLSET (TbleStart = ____)

$y = 1625.16(1.052)^8$
$y = 2512.84$
$R = obs - pred$
$= 2289 - 2512.84 = \boxed{-223.84}$

$y = 1625.16(1.052)^{14}$
$y = 3484.91$
$R = obs - pred$
$= 3771 - 3484.91 = \boxed{286.09}$

3. A certain substance has a half-life of 24 years. If a sample of 80 grams is being observed, how much will remain in 50 years? When will only 5 grams remain?

$y = ab^x$

$\frac{0.5}{1} = \frac{1(b)^{24}}{1}$

$(0.5)^{1/24} = (b^{24})^{1/24}$

$b = 0.9715$

$y = 80(0.9715)^{50}$

$y = 18.85$ grams in 50 years

$\frac{5}{80} = \frac{80(0.9715)^x}{80}$

$\frac{5}{80} = 0.9715^x$

$\log \frac{5}{80} = \log 0.9715^x$

$\frac{\log \frac{5}{80}}{\log 0.9715} = \frac{x \log 0.9715}{\log 0.9715}$

$x = 95.89$ years