FST 2-6 Notes

Topic: Quadratic Models

GOAL

Review the general quadratic equation $y = ax^2 + bx + c$, its graph, and the Quadratic Formula. Review or introduce quadratic modeling and use technology to determine the best fitting parabola.

SPUR Objectives

E Describe properties of quadratic functions.

F Find and interpret quadratic regression and models.

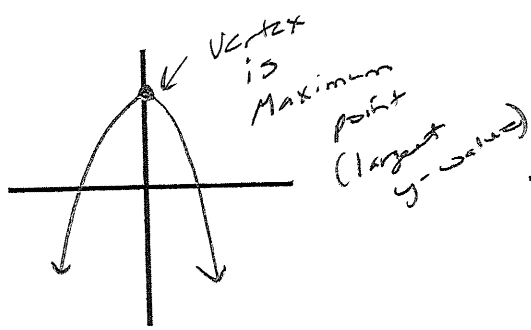
Vocabulary

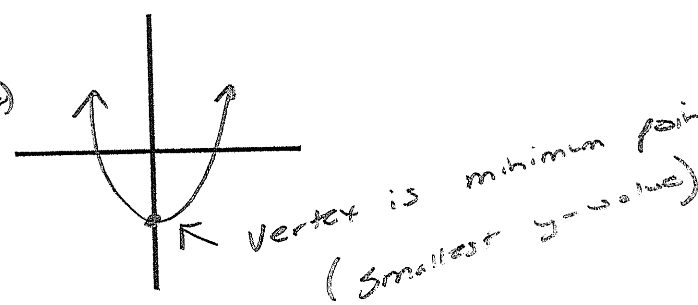
quadratic model MODELS BASED ON QUADRATIC FUNCTIONS

quadratic regression FIND EQN OF BEST FIT PARABOLA.

Properties of Quadratic Functions:

- $f(x) = ax^2 + bx + c$, where $a \neq 0$
- Graphs a parabola
- Domain: $\{x \mid x \in \mathbb{R}\}$
- (0, c) is the y-intercept





If a < 0:

Opens down

Maximum point

If a > 0:

Opens up

minimum point

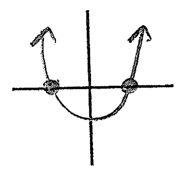
Range: $\{y \mid y \leq \text{maximum value}\}$

Range: $\{y \mid y \geq \cancel{\text{maximum}}\ \text{value}\}$ minimum x-intercepts of quadratic equation:

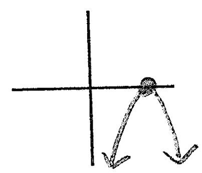
QUADRATIC FORMULA: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The discriminant, $b^2 - 4ac$, tells us information about the x-intercepts:

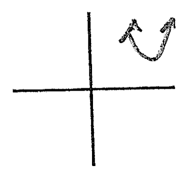
If $b^2 - 4ac > 0$, there are 2 distinct x-intercepts.



If $b^2 - 4ac = 0$, there is only 1 x-intercept.



If $b^2 - 4ac < 0$, there are no x-intercepts.

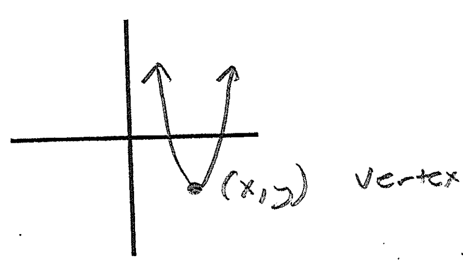


(x, y)

The vertex of the parabola is found by:

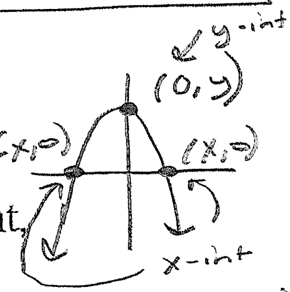
1) finding the x-coordinate, $x = -\dfrac{b}{2a}$

2) using x from step 1 to find the y-coordinate, $y = ax^2 + bx + c$



$$\left(\underbrace{\frac{-b}{2a}}_{x},\ \underbrace{a\left(\frac{-b}{2a}\right)^2 + b\left(\frac{-b}{2a}\right) + c}_{y}\right)$$

1. Consider the function f with equation $f(x) = -3x^2 - 4x + 7$.
   a. Find the x- and y-intercepts of its graph.
   b. Tell whether the parabola has a maximum or minimum point, and find its coordinates.



a) X-intercept (when y=0)
(use Quadratic Formula)

$0 = -3x^2 - 4x + 7$

$a = -3 \quad b = -4 \quad c = 7$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-3)(7)}}{2(-3)}$$

$$x = \frac{4 \pm \sqrt{100}}{-6}$$

$$x = \frac{4 \pm 10}{-6}$$

$$x = \frac{4+10}{-6} \qquad x = \frac{4-10}{-6}$$

$$x = \frac{14}{-6} \qquad x = \frac{-6}{-6}$$

$$\boxed{x = \frac{-7}{3} \qquad x = 1 \quad \text{x-ints}}$$

a) y-intercept (when x=0)

$y = -3(0)^2 - 4(0) + 7$

$y = 7$ $\boxed{(0,7) \text{ y-int}}$

b) $\boxed{\text{Maximum}}$ → opens down

Vertex: (x,y)

$$x = \frac{-b}{2a}$$

$$x = \frac{-(-4)}{2(-3)} = \frac{4}{-6} = \frac{-2}{3}$$

$$y = -3\left(-\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) + 7$$

$$y = 8\tfrac{1}{3}$$

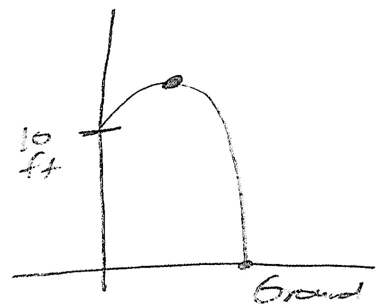
$$\boxed{\text{Vertex: } \left(-\frac{2}{3}, 8\tfrac{1}{3}\right)}$$

v_0 h_0

2. A projectile is shot from a tower 10 feet high with an upward velocity of 100 feet per second.
   a. Approximate the relationship between height h (in feet) and time t (in seconds) after the projectile is shot.
   b. How long will the projectile be in the air?

Height h of an object at time t after it has been thrown with initial velocity v_0 from an initial height h_0, $g = 32\,ft/\sec^2$

$$h = -\frac{1}{2}gt^2 + v_0 t + h_0$$



a) $h = -\frac{1}{2}(32)t^2 + 100t + 10$

$$\boxed{h = -16t^2 + 100t + 10}$$

b) $0 = -16t^2 + 100t + 10$

$a = -16 \quad b = 100 \quad c = 10$

$$X = \frac{-(100) \pm \sqrt{(100)^2 - 4(-16)(10)}}{2(-16)}$$

$$X = \frac{-100 \pm \sqrt{10640}}{-32}$$

$$X = \frac{-100 + \sqrt{10640}}{-32} \qquad X = \frac{-100 - \sqrt{10640}}{-32}$$

$X = -0.098$ sec

Not valid, Time never negative

$$\boxed{X = 6.348 \text{ sec}}$$

3. A parabola contains the points $(-0.1, -16.32)$, $(2, 3)$, and $(6, -9)$. Find its equation.

$ax^2 + bx + c = y$

By solving a system of equations:

1) Write a System of equations

$-16.32 = a(-0.1)^2 + b(-0.1) + c \rightarrow -16.32 = 0.01a - 0.1b + c$

$3 = a(2)^2 + b(2) + c \rightarrow 3 = 4a + 2b + c$

$-9 = a(6)^2 + b(6) + c \rightarrow -9 = 36a + 6b + c$

2) Write a matrix Eqn.

r × c

3×3, 3×1

$$\underbrace{\begin{bmatrix} 0.01 & -0.1 & 1 \\ 4 & 2 & 1 \\ 36 & 6 & 1 \end{bmatrix}}_{[A]} \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{[X]} = \underbrace{\begin{bmatrix} -16.32 \\ 3 \\ -9 \end{bmatrix}}_{[B]}$$

3) Solve the matrix Eqn. Write eqn. for parabola.

$X = [A]^{-1} \cdot [B]$

$$[X] = \begin{bmatrix} -2 \\ 13 \\ -15 \end{bmatrix} \begin{matrix} \rightarrow a \\ \rightarrow b \\ \rightarrow c \end{matrix}$$

$$\boxed{y = -2x^2 + 13x - 15}$$

By using quadratic regression

x	y
L1	L2
-0.1	-16.32
2	3
6	-9

STAT #1 Edit Data x values in L1 y values in L2

STAT $\rightarrow$ CALC #5

**When you identify ~~you're~~ your a, b and c values don't forget to substitute them back into the equation $y = ax^2 + bx + c$

$$\boxed{y = -2x^2 + 13x - 15}$$