

The Parabola as a Parent Function

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Topical Outline | Algebra I Outline | MathBits' Teacher Resources

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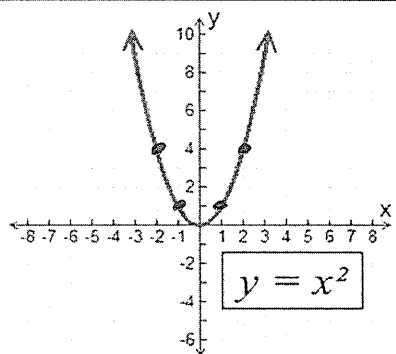
Definition: A parent function is the simplest function of a family of functions.

For the family of quadratic functions, $y = ax^2 + bx + c$, the simplest function of this form is $y = x^2$.

The "Parent" Graph:

The simplest parabola is $y = x^2$, whose graph is shown at the right. The graph passes through the origin (0,0), and is contained in Quadrants I and II.

This graph is known as the "Parent Function" for parabolas, or quadratic functions. All other parabolas, or quadratic functions, can be obtained from this graph by one or more transformations.



$V: (0,0)$

$D: \{x | x \in \mathbb{R}\}$

$R: \{y | y \geq 0\}$

x	y
-2	4
-1	1
0	0
1	1
2	4

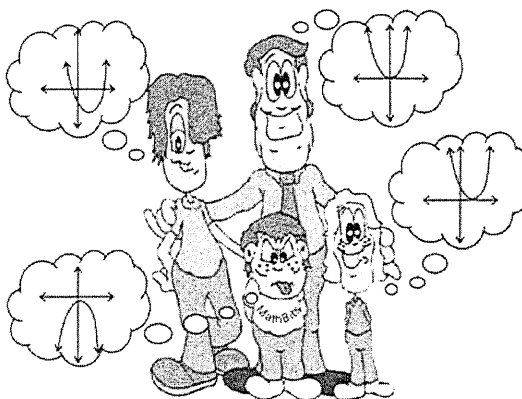
The "Children" Graphs:

The "parent" parabola can give birth to a myriad of other parabolic shapes through the process of transformations.

Brush off your memories of transformations and let's take a quick look at what is possible.

When graphing quadratic functions (parabolas), keep in mind that two forms of equations may be used:

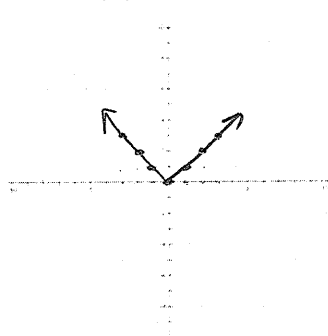
$y = ax^2 + bx + c$ or $y = a(x - h)^2 + k$



Some other "Parent" Functions

1) Absolute Value (V Shape)

$y = |x|$ $V: (0,0)$

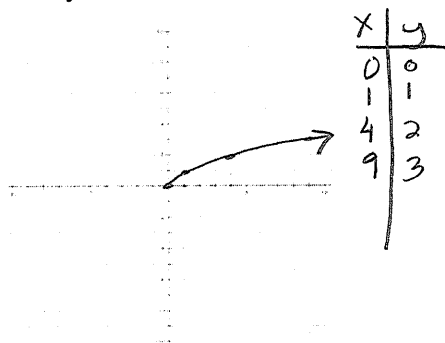


x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

$D: \{x | x \in \mathbb{R}\}$
 $R: \{y | y \geq 0\}$

2) Square Root (swoosh?)

$y = \sqrt{x}$ Start: (0,0)

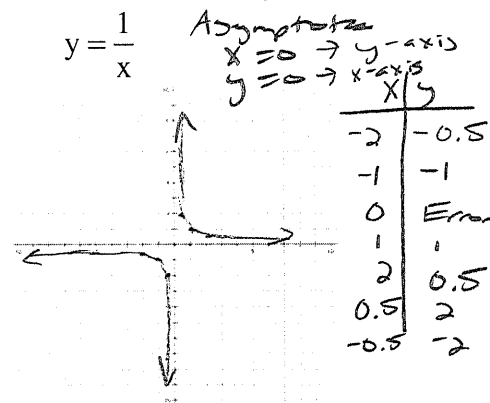


x	y
0	0
1	1
4	2
9	3

$D: \{x | x \geq 0\}$
 $R: \{y | y \geq 0\}$

3) Inverse Variation (hyperbola)

$y = \frac{1}{x}$



Asymptotes
 $x = 0 \rightarrow y\text{-axis}$
 $y = 0 \rightarrow x\text{-axis}$

x	y
-2	-0.5
-1	-1
0	Error
1	1
2	0.5
0.5	2
-0.5	-2

$D: \{x | x \neq 0\}$
 $R: \{y | y \neq 0\}$

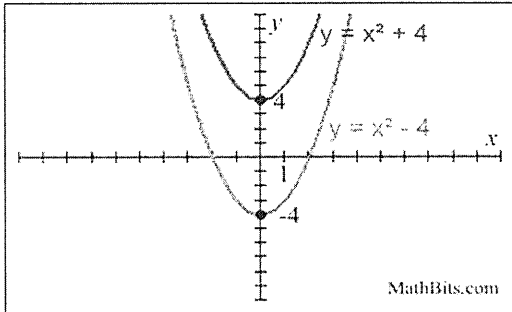
Vertical Translation

move the graph vertically - up or down

$$y = x^2 + k$$

$y = x^2 + 4$ moves the graph UP 4 units

$y = x^2 - 4$ moves the graph DOWN 4 units



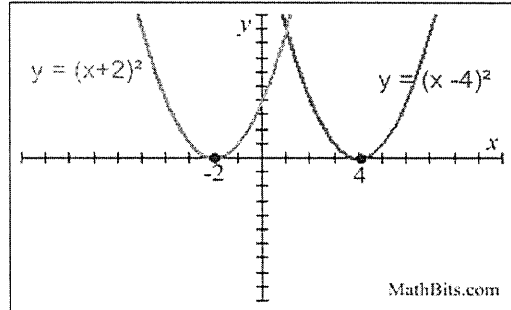
Horizontal Translation

move the graph horizontally - left or right

$$y = (x - h)^2$$

$y = (x - 4)^2$ moves the graph RIGHT 4 units

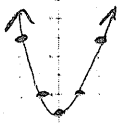
$y = (x + 2)^2$ moves the graph LEFT 2 units



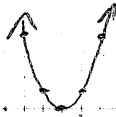
4) Graph $y = x^2 + 3$ \nearrow up 3 $V: (0, 3)$

5) Graph $y = (x - 4)^2$ \nearrow Right 4 $V: (4, 0)$

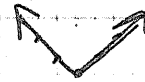
6) Graph $y = |x + 5| - 3$ \nearrow Left 5 \searrow down 3 $V: (-5, -3)$



D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 3\}$

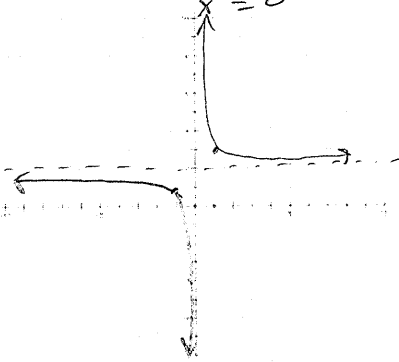


D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0\}$



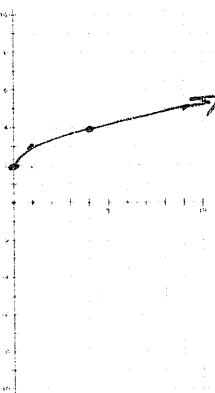
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq -3\}$

7) Graph $y = \frac{1}{x} + 2$ \nearrow up 2
Asymptotes: $y = 2$
 $x = 0$



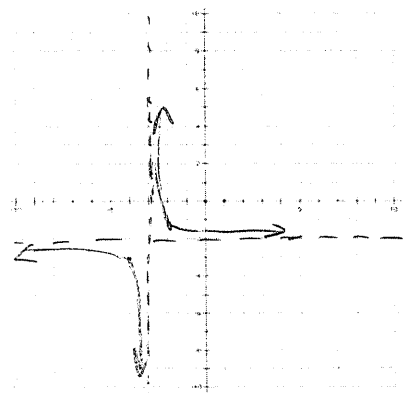
D: $\{x \mid x \neq 0\}$
R: $\{y \mid y \neq 2\}$

8) Graph $y = \sqrt{x + 2}$ \nearrow up 2
Start: (0, 2)



D: $\{x \mid x \geq 0\}$
R: $\{y \mid y \geq 2\}$

9) Graph $y = \frac{1}{x + 3} - 2$ \searrow down 2
 \nearrow Left 3



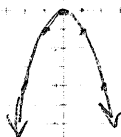
Asymptotes: $y = -2$
 $x = -3$

D: $\{x \mid x \neq -3\}$
R: $\{y \mid y \neq -2\}$

Reflection in x-axis	Reflection in y-axis
<p>flip the graph over the x-axis (negates the y-values of the coordinates)</p> <p>$y = -(x^2)$ $y = x^2$ parent graph ↗ opens up $y = -(x^2)$ parent reflected over x-axis ↘ opens down</p>	<p>flip the graph over the y-axis (negates the x-values of the coordinates)</p> <p>$y = (-x)^2$ $y = x^2$ parent graph $y = (-x)^2$ parent reflected over y-axis</p>
<p>MathBits.com</p>	<p>Since a parabola is symmetric, and the parent function is symmetric about the y-axis, this reflection creates the exact same graph again.</p> <p>MathBits.com</p>

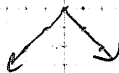
10) Graph $y = -x^2$ V: (0,0)

D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \leq 0\}$



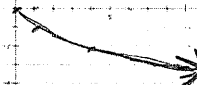
11) Graph $y = -|x|$ V: (0,0)

D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \leq 0\}$



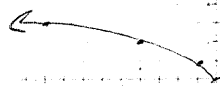
12) Graph $y = -\sqrt{x}$ Start (0,0)

D: $\{x \mid x \geq 0\}$
R: $\{y \mid y \leq 0\}$



13) Graph $y = \sqrt{-x}$

D: $\{x \mid x \leq 0\}$
R: $\{y \mid y \geq 0\}$



Reflect over y-axis
Start (0,0)

14) Graph $y = -(x+3)^2 - 2$

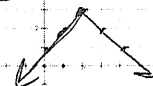
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \leq -2\}$



Vertex: (-3, -2)
Reflect over x-axis
(opens down)

15) Graph $y = -|x-2| + 3$

D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \leq 3\}$



Vertex: (2, 3)
Reflect over x-axis
(opens down)

Stretch or Compress Vertically	Stretch or Compress Horizontally
<p>stretches away from the x-axis or compresses toward the x-axis</p> $y = a \cdot x^2$ <p>$a > 1$ is a stretch; $0 < a < 1$ is a compression</p> <p>$y = x^2$ parent graph $y = \frac{1}{4}(x)^2$ vertical compression $y = 4(x)^2$ vertical stretch</p>	<p>stretches away from the y-axis or compresses toward the y-axis</p> $y = (a \cdot x)^2$ <p>$a > 1$ is a compression by factor of $1/a$; $0 < a < 1$ is a stretch by factor of $1/a$</p> <p>$y = x^2$ parent graph $y = (\frac{1}{4}x)^2$ horizontal stretch $y = (4x)^2$ horizontal compression</p>

16) Graph $y = 2x^2$ V: (0,0)
 Vert. Stretch by 2
 away from x-axis

D: $\{x \mid x \in \mathbb{R}\}$
 R: $\{y \mid y \geq 0\}$

17) Graph $y = (2x)^2$ V: (0,0)
 Horiz. compression toward y-axis by factor of $1/2$

D: $\{x \mid x \in \mathbb{R}\}$
 R: $\{y \mid y \geq 0\}$

18) Graph $y = \frac{1}{2}x^2$ V: (0,0)
 Vert. Comp. by $1/2$ toward x-axis

D: $\{x \mid x \in \mathbb{R}\}$
 R: $\{y \mid y \geq 0\}$

19) Graph $y = -3|x-2|+5$
 U.S. to 3
 Right 2
 Down 5

D: $\{x \mid x \in \mathbb{R}\}$
 R: $\{y \mid y \leq 5\}$

V: (2, 5)
 Opens Down

20) Graph $y = 2\sqrt{x+5}-3$
 U.S. to 2
 Left 5
 Down 3

D: $\{x \mid x \geq -5\}$
 R: $\{y \mid y \geq -3\}$

V: (-5, -3)

21) Graph $y = -3(x+1)^2-2$
 U.S. to 3
 Left 1
 Down 2

D: $\{x \mid x \in \mathbb{R}\}$
 R: $\{y \mid y \leq -2\}$

V: (-1, -2)
 Opens Down