FST 3-2 Notes
Topic: The Graph-Translation Theorem
GOAL:
Show that the substitution of x - h for x and y - k for y in an equation for a relation has the effect of translating a graph *h* units to the right and *k* units up.

SPUR Objectives

C Use the Graph-Translation Theorem to find transformation images.

D Describe the effects of translations on functions and their graphs.

J Apply the Graph-Translation Theorem to make or identify graphs.

Vocabulary

transformation
preimage
image
translation

Definition of Translation

A **translation** in the plane is a transformation that maps each point (x, y) onto $(x + h, y + k)$, where h and k are constant.

+h : right +k : up
−h : left −k : down

Instruction

right 12, up 25

1) Under a translation, the image of (0, 0) is (12, 25).

a) Find a rule for this translation. $(x, y) \rightarrow (x+12, y+25)$

b) Find the image (x', y') of (-2, -8) under this translation.

$(-2, -8) \rightarrow (-2+12, -8+25)$

$\boxed{(10, 17)}$

On your own:

1) Under a translation, the image of (0, 0) is (-12, 5). left 12, up 5

a) Find a rule for this translation. $(x, y) \rightarrow (x-12, y+5)$

b) Find the image of (6, -10) under this translation.

$(6, -10) \rightarrow (6-12, -10+5)$

$\boxed{(-6, -5)}$

Instruction

2) Compare the graphs of $y = x^3$ and $y - 25 = (x-12)^3$.

a) Graph it $y = (x-12)^3 + 25$

WINDOW
Xmin : −10
Xmax : 25
ymin : −10
ymax : 50

b) Describe the translation

right 12, up 25

c) Find the rule for this translation

$(x, y) \rightarrow (x+12, y+25)$

3) If the graph of $y = |x|$ is translated 2 units up and 3 units to the left, what is an equation for its image?

$y = |x + 3| + 2$

$(x, y) \rightarrow (x - 3, y + 2)$

On your own:

2) Compare the graphs of $y = x^2$, *and* $y = (x + 4.2)^2 - 5$.

Left 4.2, down 5

3) If the graph of $y = \sqrt{x}$ is translated 2 units down and 5 units to the right, what is an equation for its image?

$y = \sqrt{x - 5} - 2$

Instruction

a) If the graph of $y = \sqrt{x}$ is translated 2 units down and 5 units to the right, what is the translation rule?

$y = \sqrt{x - 5} - 2$

$(x, y) \rightarrow (x + 5, y - 2)$

Graph Translation Theorem

Given a preimage graph described by a sentence in x and y, the following two processes yield the same image:

(1) replacing x by $x - h$ and y by $y - k$ in the sentence;

(2) applying the translation $(x, y) \rightarrow (x + h, y + k)$ to the preimage graph.

- For T (x, y) = (x + h, y + k) the image of y = f(x) is

  $y - k = f(x - h)$ (+k)

  or $y = f(x - h) + k$ (−k)

Example: If the graph of $y = -\dfrac{1}{2x^2}$ is translated 8 units up and 17 units to the left, what is an equation for its image?

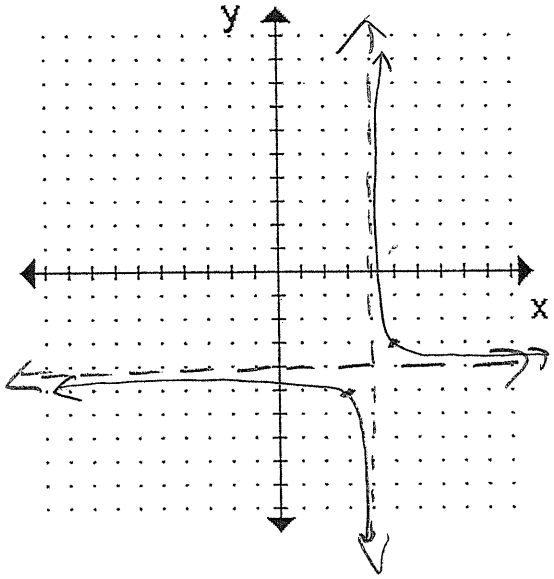
$y = -\dfrac{1}{2(x+17)^2} + 8$

↑ 8

↑ L17

Example: Sketch the graph of $y = \dfrac{1}{(x-4)} - 4$ down 4 Asy: $y = -4$

↑ R 4 Asy: $x = 4$



D: $\{x \mid x \neq 4\}$

R: $\{y \mid y \neq -4\}$

A good graph meets the following criteria:

- Axes are labeled appropriately, with the scales shown.
- The characteristics of the graph can be seen (appropriate window).
  - -asymptotes
  - -discontinutities
  - -changes in direction
- The intercepts are shown.