FST 3-4 Notes
Topic: Symmetries of Graphs
GOAL: Review the ideas of reflection and rotation symmetry, apply them to graphs of functions, and to the ideas of even and odd functions.

SPUR Objectives

D Describe the effects of translations on functions and their graphs.

E Describe and identify symmetries and asymptotes of graphs.

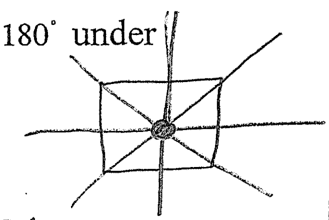
I Recognize functions and their properties from their graphs.

Vocabulary

reflection-symmetric MAPPED ONTO ITSELF BY REFLECTION OVER SOME LINE
axis of symmetry - REFLECTION LINE
line of symmetry
symmetric about a point - MAPPED ONTO ITSELF BY 180° ROTATION
point symmetry —
even function -> SYMMETRIC OVER Y-AXIS $f(-x) = f(x)$
odd function - SYMMETRIC ABOUT ORIGIN $f(-x) = -f(x)$

The **line of symmetry** can be any line in the plane

Center of symmetry for a figure = the center of rotation of 180° under which the figure is mapped onto itself



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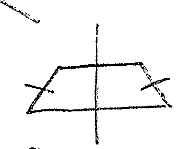
Warm-Up

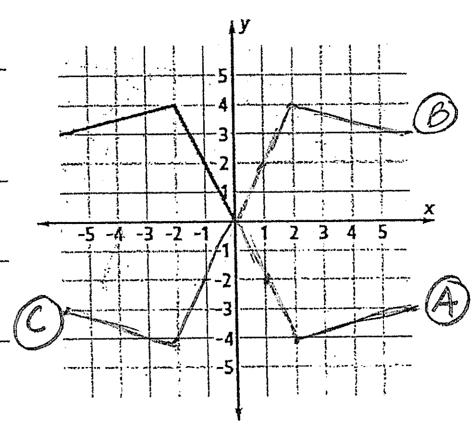
1. How many symmetry lines does a square have? 4
2. How many centers of symmetry does a square have? 1
3. How many symmetry lines does an isosceles trapezoid have? 1
4. How many centers of symmetry does an isosceles trapezoid have? 0



Activity 1

The diagram at the right shows half of a graph.

Step 1	Copy the diagram. Draw the other half of the graph so that the result is point-symmetric about the origin. Label this half *A*.
Step 2	Draw the other half of the original graph so that the result is symmetric with respect to the *y*-axis. Label this half *B*.
Step 3	Draw the other half of the original graph so that it is symmetric over the *x*-axis. Label the graph *C*.
Step 4	What symmetries does the union of graphs *A*, *B*, and *C* and the original graph possess?



The reflection image of (x, y) over the x-axis is $(x, -y)$

The reflection image of (x, y) over the y-axis is $(-x, y)$

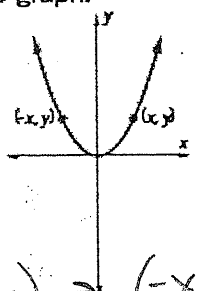
The image of (x, y) under a rotation of 180° about the origin is $(-x, -y)$

The union is reflection symmetric over both axes and point-symmetric about the origin.

Symmetries of Graphs

Theorem (Symmetry over y-axis)

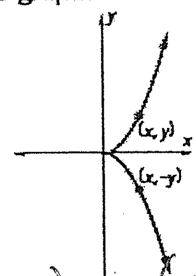
A graph is symmetric with respect to the y-axis if and only if for every point (x, y) on the graph, (−x, y) is also on the graph.



$(x,y) \to (-x,y)$
x changes
y the same

Theorem (Symmetry over x-axis)

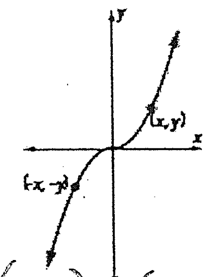
A graph is symmetric with respect to the x-axis if and only if for every point (x, y) on the graph, (x, −y) is also on the graph.



$(x,y) \to (x,-y)$
x the same
y changes

Theorem (Symmetry about the Origin)

A graph is symmetric to the origin if and only if for every point (x, y) on the graph, (−x, −y) is also on the graph.



$(x,y) \to (-x,-y)$
x changes
y changes

Proving that a graph has symmetry:

Example 1: Prove that the graph of $y = \sqrt{36 - x^2}$ is symmetric to the y-axis.

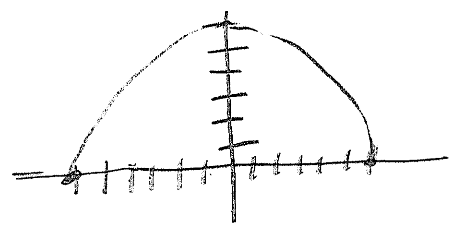
$(-x)^2 = x^2$

y-axis : $(x,y) \to (-x,y)$

$\sqrt{36 - x^2}$ $\qquad \sqrt{36 - (-x)^2}$

$\sqrt{36 - x^2} = \sqrt{36 - x^2}$

yes, symmetric to y-axis



Is $y = \sqrt{36 - x^2}$ symmetric with respect to the x-axis? The origin?

x-axis : $(x,y) \to (x,-y)$

$y = \sqrt{36 - x^2}$ $\qquad -y = \sqrt{36 - x^2}$

$y = \sqrt{36 - x^2} \neq y = -\sqrt{36 - x^2}$

No, not symmetric to x-axis

origin: $(x,y) \to (-x,-y)$

$y = \sqrt{36 - x^2}$ $\qquad -y = \sqrt{36 - (-x)^2}$

$y = \sqrt{36 - x^2} \neq y = -\sqrt{36 - x^2}$

Not symmetric to origin

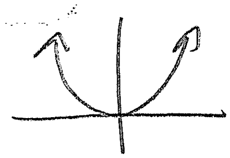
Even and Odd Functions

Definition of Even Function

A function is an **even function** if and only if for all values of x in its domain, $f(-x) = f(x)$.

Transformation
$(x, y) \rightarrow (-x, y)$

* An even function has symmetry with respect to the y-axis.



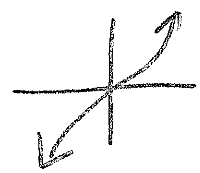
Ex: $y = x^2$

Definition of Odd Function

A function f is an **odd function** if and only if for all values of x in its domain, $f(-x) = -f(x)$.

Transformation
$(x, y) \rightarrow (-x, -y)$

* An odd function has symmetry with respect to the origin.



Ex: $y = x^3$

Example 2: Determine (algebraically, not graphically) whether the function

$f(x) = x^3 - 5x$ is odd, even, or neither.

ODD: $f(-x) = -f(x)$

$(-x)^3 - 5(-x) \qquad -(x^3 - 5x)$

$-x^3 + 5x = -x^2 + 5x$

$\boxed{\text{yes, odd function}}$

EVEN: $f(-x) = f(x)$

$(-x)^3 - 5(-x) \qquad x^3 - 5x$

$-x^3 + 5x \neq x^3 - 5x$

$\boxed{\text{No, not even function}}$

V. Stretch by 3

↑ 9.5

Example 3: Consider the function H with $y = H(x) = \dfrac{3}{x-8} + 9.5$

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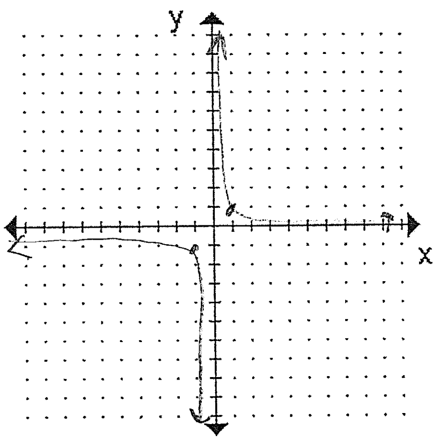
a. Give equations for the **asymptotes** of its graph.

* Hint: Identify the parent function first! $y = \frac{1}{x}$

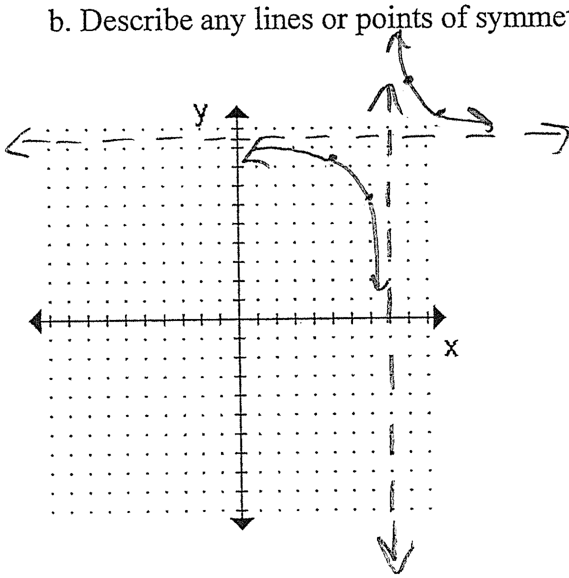


Asymptotes

$X = 0$ $y = 0$

Point of Symmetry: $(0,0)$

b. Describe any lines or points of symmetry.



Asymptotes

$X = 8$ $y = 9.5$

Point of symmetry: $(8, 9.5)$