① $f(x) = x + 3$ $g(x) = \sqrt{9-x^2}$

$$\boxed{f \circ g = f(g(x))}$$

Domain of $g(x) = \sqrt{9-x^2}$

$$9 - x^2 \geq 0$$
$$9 \geq x^2$$
$$\sqrt{x^2} \leq \sqrt{9}$$
$$x \leq 3 \qquad x \geq -3$$

$$\boxed{f(g(x)) = \sqrt{9-x^2} + 3}$$

$$9 - x^2 \geq 0$$
$$9 \geq x^2$$
$$\sqrt{x^2} \leq \sqrt{9}$$
$$x \leq 3 \qquad x \geq -3$$

$$\boxed{\text{Domain}: \{x \mid x \leq 3, x \geq -3\}}$$
$$\boxed{\text{or } \{x \mid -3 \leq x \leq 3\}}$$

$$\boxed{g \circ f = g(f(x))}$$

Domain of $f(x) = x + 3$ $\{x \mid x \in \mathbb{R}\}$

$$g(f(x)) = \sqrt{9 - (x+3)^2}$$
$$= \sqrt{9 - (x^2 + 6x + 9)}$$
$$= \sqrt{9 - x^2 - 6x - 9}$$

$$\boxed{g(f(x)) = \sqrt{-x^2 - 6x}}$$

$$-x^2 - 6x \geq 0$$
$$-x(x+6) \geq 0$$

$$\frac{-x \geq 0}{-1} \quad \frac{}{-1} \qquad \begin{array}{c} x + 6 \geq 0 \\ -6 \quad -6 \end{array}$$

$$x \leq 0 \qquad\qquad x \geq -6$$

$$\boxed{\text{Domain}: \{x \mid x \leq 0, x \geq -6\}}$$
$$\boxed{\text{or } \{x \mid -6 \leq x \leq 0\}}$$

② $f(x) = \sqrt{x+3}$ $\qquad$ $g(x) = 2x-5$

$$\boxed{f \circ g = f(g(x))}$$

Domain of $g(x) = 2x-5$ $\qquad$ $\{x \mid x \in \mathbb{R}\}$

$f(g(x)) = \sqrt{2x-5} +3$

$$\boxed{f(g(x)) = \sqrt{2x-2}}$$

$$2x-2 \geq 0$$
$$+2 \quad +2$$
$$\frac{2x}{2} \geq \frac{2}{2}$$
$$x \geq 1$$

$$\boxed{\text{Domain } \{x \mid x \geq 1\}}$$

$$\boxed{g \circ f = g(f(x))}$$

Domain of $f(x) = \sqrt{x+3}$

$$x+3 \geq 0$$
$$-3 \quad -3$$
$$x \geq -3 \qquad \{x \mid x \geq -3\}$$

$$\boxed{g(f(x)) = 2(\sqrt{x+3}) -5}$$

$$x+3 \geq 0$$
$$-3 \quad -3$$
$$x \geq -3$$

$$\boxed{\text{Domain: } \{x \mid x \geq -3\}}$$

③ $f(x) = \dfrac{-3}{x}$ $g(x) = \dfrac{x}{x-2}$

$\boxed{f \circ g = f(g(x))}$

Domain of $g(x) = \dfrac{x}{x-2}$

$x - 2 \neq 0$
$ +2 \quad +2 \qquad \{x \mid x \neq 2\}$
$x \neq 2$

$f(g(x)) = \dfrac{-3}{\frac{x}{x-2}} \cdot \dfrac{x-2}{x} = \boxed{\dfrac{-3(x-2)}{x} = f(g(x))}$

$x \neq 0$

$\boxed{\text{Domain} : \{x \mid x \neq 2, \; x \neq 0\}}$

$\boxed{g \circ f = g(f(x))}$

Domain of $f(x) = x \neq 0$ $\qquad \{x \mid x \neq 0\}$

$g(f(x)) = \dfrac{\frac{-3}{x}}{\frac{-3}{x} - 2} = \dfrac{\frac{-3}{x}}{\frac{-3}{x} - \frac{2x}{x}} = \dfrac{\frac{-3}{x}}{\frac{-3-2x}{x}} \cdot \dfrac{x}{-3-2x}$

$\boxed{g(f(x)) = \dfrac{-3}{-3-2x}}$

$-3 - 2x \neq 0$
$+3 \qquad\quad +3$

$\dfrac{2x}{2} \neq \dfrac{3}{2}$

$x \neq -\dfrac{3}{2}$

$\boxed{D: \{x \mid x \neq 0, \; x \neq -3/2\}}$

④ $f(x) = x^2 + 2$ $g(x) = \sqrt{x-5}$

$$f \circ g = f(g(x))$$

Domain of $g(x) = \sqrt{x-5}$

$$x - 5 \geq 0$$
$$+5 \quad +5$$
$$x \geq 5 \qquad \{x \mid x \geq 5\}$$

$$f(g(x)) = \left(\sqrt{x-5}\right)^2 + 2$$

$$= x - 5 + 2$$

$$f(g(x)) = x - 3 \qquad \text{All Real}$$

$$\boxed{\text{Domain: } \{x \mid x \geq 5\}}$$

$$g \circ f = g(f(x))$$

Domain of $f(x) = x^2 + 2$ $\qquad D: \{x \mid x \in \mathbb{R}\}$

$$g(f(x)) = \sqrt{x^2 + 2 - 5}$$
$$\boxed{g(f(x)) = \sqrt{x^2 - 3}}$$

$$x^2 - 3 \geq 0$$
$$+3 \quad +3$$
$$\sqrt{x^2} \geq \sqrt{3}$$
$$x \geq \sqrt{3} \qquad x \leq \sqrt{3}$$

$$\boxed{\text{Domain: } \{x \mid x \geq \sqrt{3}, \ x \leq -\sqrt{3}\}}$$

⑤ $f(x) = \dfrac{2}{x-3}$ $\qquad$ $g(x) = \dfrac{5}{x+2}$

$\boxed{f \circ g = f(g(x))}$

Domain of $g(x) = \dfrac{5}{x+2}$

$$x + 2 \neq 0$$
$$\underline{-2 \quad -2}$$
$$x \neq -2 \qquad \{x \mid x \neq -2\}$$

$f(g(x)) = \dfrac{2}{\dfrac{5}{x+2} - \dfrac{3(x+2)}{x+2}} = \dfrac{2}{\dfrac{5 - 3(x+2)}{x+2}} \cdot \dfrac{\frac{x+2}{5-3(x+2)}}{}$

$= \dfrac{2(x+2)}{5 - 3(x+2)} = \dfrac{2(x+2)}{5 - 3x - 6} = \boxed{\dfrac{2(x+2)}{-3x - 1} = f(g(x))}$

$$-3x - 1 \neq 0$$
$$\underline{+1 \quad +1}$$
$$\dfrac{-3x}{-3} \neq \dfrac{1}{3}$$
$$x \neq -\dfrac{1}{3}$$

$\boxed{\text{Domain } \{x \mid x \neq -\frac{1}{3}, x \neq -2\}}$

$\boxed{g \circ f = g(f(x))}$

Domain of $f(x) = \dfrac{2}{x-3}$ $\qquad$ $x - 3 \neq 0$
$$\underline{+3 \quad +3}$$
$$x \neq 3 \qquad \{x \mid x \neq 3\}$$

$g(f(x)) = \dfrac{5}{\dfrac{2}{x-3} + \dfrac{2(x-3)}{x-3}} = \dfrac{5}{\dfrac{2 + 2x - 6}{x-3}} = \dfrac{5}{\dfrac{2x - 4}{x-3}} \cdot \dfrac{x-3}{2x-4}$

$\boxed{g(f(x)) = \dfrac{5(x-3)}{2x - 4}}$

$$2x - 4 \neq 0$$
$$\underline{+4 \quad +4}$$
$$\dfrac{2x}{2} \neq \dfrac{4}{2}$$
$$x \neq 2$$

$\boxed{D: \{x \mid x \neq 2, x \neq 3\}}$

⑥ $f(x) = \dfrac{1}{\sqrt[3]{x} - 2}$ $g(x) = x^2 - 3$

$\boxed{f \circ g = f(g(x))}$

Domain of $g(x) = x^2 - 3$ $\{x \mid x \in \mathbb{R}\}$

$\boxed{f(g(x)) = \dfrac{1}{\sqrt[3]{x^2 - 3} - 2}}$

$\sqrt[3]{x^2 - 3} - 2 \neq 0$

$\phantom{\sqrt[3]{x^2-3}}\; +2 \qquad +2$

$\left(\sqrt[3]{x^2 - 3}\right)^3 \neq (2)^3$

$x^2 - 3 \neq 8$

$\; +3 \qquad +3$

$\sqrt{x^2} \neq \sqrt{11} \qquad x \neq \pm\sqrt{11}$

$\boxed{D: \{x \mid x \neq \pm\sqrt{11}\}}$

$\boxed{g \circ f = g(f(x))}$

Domain of $f(x) = \sqrt[3]{x} - 2$ $\{x \mid x \in \mathbb{R}\}$

$\boxed{g(f(x)) = \left(\dfrac{1}{\sqrt[3]{x} - 2}\right)^2 - 3}$

$\sqrt[3]{x} - 2 \neq 0$

$\phantom{\sqrt[3]{x}}\; +2 \qquad +2$

$\left(\sqrt[3]{x}\right)^3 \neq (2)^3$

$x \neq 8$ $\boxed{D: \{x \mid x \neq 8\}}$