FST 3-8 Notes
Topic: Inverses of Functions
GOAL:
Define inverse of a function, discuss how to determine an inverse of a function from its graph or by graphing, and then examine the skill of finding an equation of an inverse.

SPUR Objectives

B Find inverses of functions.

F Identify properties of inverses of functions.

I Recognize functions and their properties from their graphs.

K Graph inverses of functions.

Vocabulary

Inverse of a function
Identity function

Mental Math

What operation undoes each action?

a. adding $\frac{2}{3}$ to a number

b. multiplying a number by $\frac{\pi}{2}$

c. squaring a positive number

Inverse of a function: the relation in which the components of all ordered pairs of the function are switched

* every function has an inverse, but not all inverses are functions.

Notation: the inverse of f(x) is denoted $f^{-1}(x)$

* $f^{-1}(x) \neq \dfrac{1}{f(x)}$ even though $x^{-1} = \dfrac{1}{x}$

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Example 1: Let h = {(1, 1), (2, 4), (3, 9), (4, 16)}

a) Is h a function? Explain.

YES - NO REPEAT X-VALUES

b) Describe the inverse of h. ☆ Switch x and y

$h^{-1} = \{(1,1), (4,2), (9,3), (16,4)\}$

c) Is the inverse a function? Explain.

YES - NO REPEAT X-VALUES

d) Describe h and its inverse in words.

h → Square x to get y

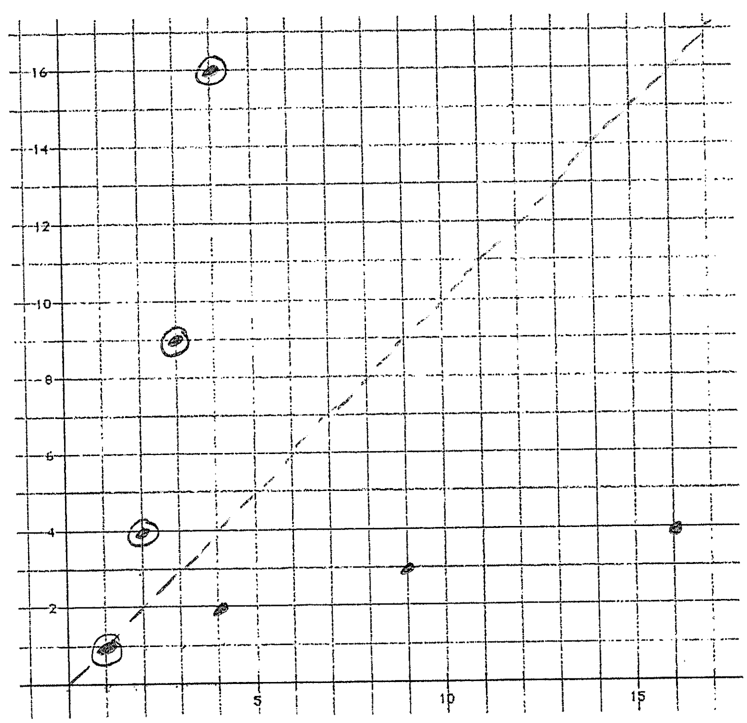
h^{-1} → Square root y to get x

e) Plot the points for h and its inverse.
What do you notice?

h = ⊙

h^{-1} = •

A Function and its inverse reflect over the line y = x



Example 2: Consider the function $y = 2(x+5)^2 - 1$.

a) Describe the graph of the function.

Parabola, Left 5, Down 1, V. Stretch by 2

Vertex $(-5,-1)$, opens up

b) Give an equation for the inverse of the function.

① SWITCH X AND Y

$x = 2(y+5)^2 - 1$

$+1 \qquad\qquad +1$

$\dfrac{x+1}{2} = \dfrac{2(y+5)^2}{2}$

$\pm\sqrt{\dfrac{x+1}{2}} = \sqrt{(y+5)^2}$

② SOLVE FOR Y

$y + 5 = \pm\sqrt{\dfrac{x+1}{2}}$

$-5 \qquad\qquad -5$

$\boxed{y^{-1} = \pm\sqrt{\dfrac{x+1}{2}} - 5}$

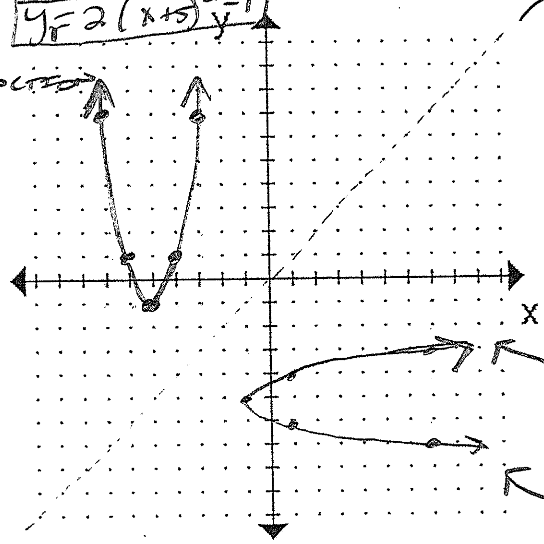
c) Based on your answer to Part a, describe the graph of the inverse of the function. Is the inverse a function?

$\boxed{y_1 = 2(x+5)^2 - 1}$

YES, FUNCTION

x	y
-7	7
-6	1
-5	-1
-4	1
-3	7

$\boxed{y_4 = x}$



Inverse – Not a function, FAILS VLT

x	y
7	-7
1	-6
-1	-5
1	-4
7	-3

$\boxed{y_2 = \sqrt{\dfrac{x+1}{2}} - 5}$

$\boxed{y_3 = -\sqrt{\dfrac{x+1}{2}} - 5}$

☆ LOOK AT TABLE ON CALC

HYPERBOLA, L 8, ↓ 1, V. Stretch of 5

Example 3: Give an equation for the inverse of the function with equation $y = \dfrac{5}{x+8} - 1$

① SWITCH X AND Y ② SOLVE FOR Y

$x = \dfrac{5}{y+8} - 1$

$+1 \qquad +1$

$\dfrac{x+1}{1} = \dfrac{5}{y+8}$

$\dfrac{5}{x+1} = \dfrac{(x+1)(y+8)}{x+1}$

$\dfrac{5}{x+1} = y+8$

$-8 \qquad -8$

$\boxed{y^{-1} = \dfrac{5}{x+1} - 8}$

HYPERBOLA, R 1, ↓ 8, V. STRETCH BY 5

b) Is the inverse a function?

YES - PASSES VLT

Inverse Functions and Composite Functions

Inverses of Functions Theorem

Given any two functions f and g, f and g are inverse functions if and only if $f(g(x)) = x$ for all x in the domain of g, and $g(f(x)) = x$ for all x in the domain of f.

UPOU

Example 4: Use the inverse of Functions Theorem to determine whether f and g are inverses.

$$f(x) = \frac{3x+1}{5-x} \qquad g(x) = \frac{5x-1}{x+3}$$

$$f(g(x)) = \frac{3\left(\frac{5x-1}{x+3}\right) + \frac{1}{1}\frac{(x+3)}{(x+3)}}{\frac{(x+3)}{(x+3)}\frac{5}{1} - \frac{5x-1}{x+3}} = \frac{\frac{15x-3+x+3}{\cancel{x+3}}}{\frac{5x+15-5x+1}{x+3}} \cdot \frac{\cancel{x+3}}{5x+15-\cancel{5x}+1}$$

$$= \frac{16x}{16} = \boxed{x}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, they are inverse functions

$$g(f(x)) = \frac{5\left(\frac{3x+1}{5-x}\right) - \frac{1}{1}\frac{(5-x)}{(5-x)}}{\frac{3x+1}{5-x} + \frac{3}{1}\frac{(5-x)}{(5-x)}} \quad \frac{\frac{15x+5-5+1x}{\cancel{5-x}}}{\frac{3x+1+15-3x}{5-x}} \cdot \frac{\cancel{5-x}}{3x+1+15-3x}$$

$$= \frac{16x}{16} = \boxed{x}$$