FST 6-2 Notes
Topic: Principles of Probability
<u>GOAL</u>
Develop two forms of Addition Counting Principle from basic principles of probability, and use them to develop theorems for finding probabilities of unions of events (mutually exclusive or not), and probabilities of complementary events.

SPUR Objectives

A Compute probabilities of events in various contexts.

D Compute probabilities using the General and Mutually Exclusive Forms of the Probability of the Union of Events and the Probability of Complements.

Vocabulary

union of sets
disjoint sets
mutually exclusive sets
intersection of sets
complementary events
complement of *A*, not *A*

Consider the two experiments.
What would be their possible outcomes?

Experiment 1: A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing a 5 or a king?
Probabilities: IN A DECK, THERE ARE 4 "5's" AND 4 "KINGS"

$P(5 \text{ or } K) = \frac{8}{52}$

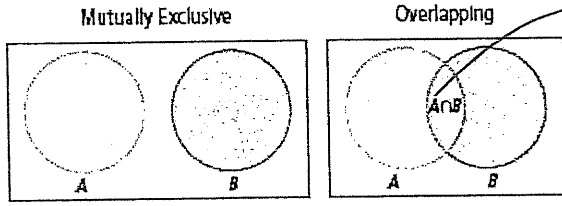
- 5
- KING

Experiment 2: A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing a club or a king?
Probabilities: IN A DECK, THERE ARE 13 CLUBS, 4 KINGS, AND ONE KING OF CLUBS

$P(\text{CLUB or } K) = \underset{\text{CLUBS}}{13} + \underset{\text{KING}}{4} - \underset{\text{KING OF CLUBS}}{1} = \frac{16}{52}$

- CLUB
- KING
- KING AND CLUB

What is the difference between the two experiments?
OVERLAP IN EXP. 2

Set language	Disjoint	Intersecting
Event language	Mutually Exclusive	Overlapping
Venn diagram		
Symbols	$A \cap B = \emptyset$	$A \cap B \neq \emptyset$
	EXP 1	EXP 2

KING OF CLUBS

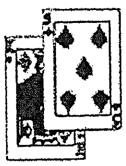
Mutually Exclusive

Two events are $\boxed{\textbf{mutually exclusive}}$ if they cannot occur at the same time (i.e., they have no outcomes in common). $\boxed{A \cup B,}$ **the union of set A and B.**

Addition Counting Principle (Mutually Exclusive Form)

If two finite sets A and B are mutually exclusive, then $N(A \cup B) = N(A) + N(B)$.

Experiment 1: A single card is chosen at random form a standard deck of 52 playing cards. Suppose you want to choose a 5 or a king. How many outcomes are there?



5 : 4
K : 4

4 + 4 = 8 outcomes

Theorem (Probability of the Union of Mutually Exclusive Events)

If A and B are mutually exclusive events in the same finite sample space, then $P(A \cup B) = P(A) + P(B)$.

What is the probability of choosing a 5 or a king?

$$P(5 \text{ or } K) = P(5) + P(K) = \frac{4}{52} + \frac{4}{52} = \boxed{\frac{8}{52}}$$

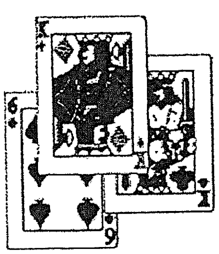
Overlapping Events – Intersection

Occurs when two events are not mutually exclusive; they have outcomes in common.
$\boxed{A \cap B,}$ **the intersection of sets A and B.**

Addition Counting Principle (General Form)

For any finite sets A and B, $N(A \cup B) = N(A) + N(B) - N(A \cap B)$.

Experiment 2: A single card is chosen at random from a standard deck of 52 playing cards. Suppose you want a club or a king. How many outcomes are there?



CLUB : 13
KING : 4
SHARE : 1
(KING OF CLUBS)

13 + 4 − 1 = 16

Theorem (Probability of the Union of Events, General Form)

If A and B are any events in the same finite sample space, then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

What is the probability of choosing a club or king?

$$P(\text{CLUB OR KING}) = P(\text{CLUB}) + P(\text{KING}) - P(\text{SHARE})$$

$$\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \boxed{\frac{16}{52}}$$

Example 1: A pair of six-sided dice is thrown. If the dice are fair, what is the probability that the dice show a sum of 7 or 11? OR → (+)

Sum of 7: {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}
Sum of 11: {(5,6), (6,5)}

$P(7 \text{ or } 11) = P(7) + P(11)$

$\frac{6}{36} + \frac{2}{36} = \boxed{\frac{8}{36} = \frac{2}{9}}$

• Mutually Exclusive
• No overlap

Example 2: What is the probability that the dice show doubles or a sum of 8?

Doubles: {(1,1), (2,2), (3,3), $\boxed{(4,4)}$, (5,5), (6,6)}
Sum of 8: {(2,6), (3,5), $\boxed{(4,4)}$, (5,3), (6,2)}
OVERLAP – NOT MUTUALLY EXCLUSIVE

$P(\text{Doubles or } 8) = P(\text{Doubles}) + P(8) - P(\text{Shared})$
$= \frac{6}{36} + \frac{5}{36} - \frac{1}{36} = \boxed{\frac{10}{36} = \frac{5}{18}}$

Complementary Events
Events that are mutually exclusive and their union is the entire sample space

Experiment	Sample Space	Event	Complement	
tossing a coin	{heads, tails}	{tails}	{heads}	Not tails
tossing two coins	{HH, HT, TH, TT}	getting no heads {TT}	getting 1 or 2 heads {HH, HT, TH}	Not getting no heads
picking an integer from 1 to 100	$\{n \in \mathbf{Z} : 1 \le n \le 100\}$	picking a prime number	picking 1 or a composite number	Not picking prime #

Not A

The **complement of an event A** is called **not A.**

Theorem (Probability of Complements)

If A is any event, then $P(\text{not } A) = 1 - P(A)$.

Let $P(\text{wearing jeans}) = 5/7$

Then $P(\text{not wearing jeans}) = 1 - \frac{5}{7} = \frac{7}{7} - \frac{5}{7} = \boxed{\frac{2}{7}}$

Example 3: A pair of six-sided dice is thrown. If the dice are fair, what is the probability that the dice show a product between 7 and 31?

1 − P(Not between) = P(Between)

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Not btwn = 15

btwn = 21

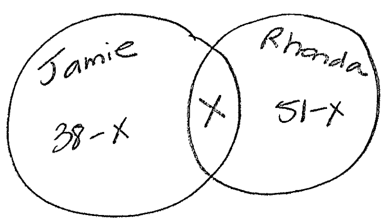
$\boxed{\frac{21}{36}}$

$1 - \frac{15}{36}$

$\frac{36}{36} - \frac{15}{36} = \boxed{\frac{21}{36}}$

Example 4: Jamie and Rhonda had to take a make up test after school. Their teacher told them they could come in no earlier than 3:00 and leave no later than 4:00. Jamie took 38 minutes on the test and Rhonda 51 minutes. For at least how many minutes were Jamie and Rhonda taking the test at the same time?

X = overlap



$38 - X + X + 51 - X = 60$

$89 - X = 60$

$-60 \quad -X \qquad -60 \quad -X$

$\boxed{27 = X}$

minutes