FST 6-4 Notes
Topic: Counting Strings without replacement
GOAL
Cover the most general type of permutation without replacement, and present the familiar formulas for $_nP_r$.

SPUR Objectives

B Find the number of strings without replacement.
C Evaluate expressions using factorials.
I Calculate probabilities in real situations.
J Use permutations to find the number of ways of arranging objects.

Vocabulary

permutation – arrangement where order matters
permutation of n objects taken r at a time, $_nP_r$

Warm-Up

Ten permutations of the letters AELST are legal words in the game of Scrabble® using the *Official Scrabble® Players Dictionary, 4th Edition.*

1. Name as many as you can.

STALE SLATE
STEAL TEALS – a teal is a small freshwater duck
LEAST TAELS - weight used in China
TALES STELA - upright stone slab
TESLA SETAL - plural of seta - stiff hair/bristle on organisms

2. If the letters AELST are ordered at random, what is the probability that the ordering forms a legal word in Scrabble®?

$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 120$ total permutations

$P(\text{Legal Word}) = \dfrac{10}{120} = \dfrac{1}{12}$

Example 1
In volleyball, a team plays 6 players at a time, three at the net and three behind. How many different ways are there to arrange three at the net from the six who are playing?

$_nP_r = {}_6P_3 = 120$

$\underline{6} \cdot \underline{5} \cdot \underline{4} = 120$

As in the last example, arrangements with OUT replacement are called permutations

In the last example, 6 players were taken 3 at a time = ${}_6P_3 = 120$

The number of permutation of n objects taken r at a time = ${}_nP_r$

Theorem (Alternate Formula for ${}_nP_r$)

$${}_nP_r = \frac{n!}{(n-r)!}$$

${}_nP_r$

$${}_6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}} = 120$$

Example 2

An art gallery has 12 paintings by Renaissance painters but only enough room to show 8 of them. In how many ways can they arrange 8 of the 12 paintings in the 8 places they have for them?

$${}_{12}P_8 = \frac{12!}{(12-8)!} = \frac{12!}{4!} = 19{,}958{,}400$$

$$= 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$$

Permutations of *n* Objects Taken *n* at a Time.

Corollary (Formula for ${}_nP_n$)

There are $n!$ permutations of n different elements. That is,

$${}_nP_n = n!.$$

Definition

$$0! = 1$$

$${}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{120}{1} = 120$$

Example 3

Tired of hearing his 17 students argue about who gets to be first in line, first-grade teacher Kerry Okle decides to put the students in a different order every time they get in line. He decides to list all the possible orders so that he will not duplicate any of them. If it takes him 20 seconds to write each order, how long will it take him to finish his list?

$_{17}P_{17} = \; = 17! = 3.56 \times 10^{14} \times 20\text{ sec} = 7.12 \times 10^{15}\text{ sec}$

$$\frac{1\text{ yr}}{365\text{ days}} \cdot \frac{1\text{ day}}{24\text{ hr}} \cdot \frac{1\text{ hr}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} \cdot \frac{7.12 \times 10^{15}\text{ sec}}{}$$

$= \boxed{225773718.9\text{ years}}$

If he decided to put the 6 girls in his class first and then the 11 boys, how many different orders would he have?

$_6P_6 \cdot {}_{11}P_{11}$

$6! \cdot 11! = \boxed{2.87 \times 10^{10}}$