FST 6-6 Notes
Topic: Conditional Probability
GOAL
Introduce the language and notation of conditional probability and apply conditional probability to situations where the answers are not at all obvious.

SPUR Objectives

D Calculate probabilities using the definition of conditional probability.

I Calculate probabilities in real situations.

Vocabulary

conditional probability of an event, $P(B|A)$

Warm-up
Suppose 60% of the singers in a school play are in the school choir. In the school as a whole, suppose 10% of the students are in the choir and 5% are in the school play. Finally, suppose there are 600 students in the school.

a. If a student in the play is randomly chosen, what is the probability that the student is in the choir?

$(0.10)(600) = 60$ students in choir
$(0.05)(600) = 30$ students in play
$(0.60)(30) = 18$ in play and choir

$\dfrac{18 \text{ in play and choir}}{30 \text{ total in play}} = \boxed{60\%}$

b. If a student in the choir is randomly chosen, what is the probability that student is in the play?

$\dfrac{18 \text{ play and choir}}{60 \text{ total in choir}} = \boxed{30\%}$

c. If a student in the school is randomly chosen, what is the probability that the student is in both the play and the choir?

$\dfrac{18 \text{ play + choir}}{600 \text{ students in school}} = \boxed{3\%}$

Conditional Probability

What is the probability that a random student in our school walked to school today?

Let w = walked
S = students

$P(w) = \dfrac{N(w)}{N(S)} = \dfrac{\#\text{ of walkers}}{\#\text{ of students}}$

What is the probability that a student who lives over 1 mile from school walked to school today?

P(walked given of 1 mile from school)

$P(w|m) = \dfrac{P(w \cap m)}{P(m)} = \dfrac{P(\text{Both})}{P(m)}$

Titanic Table 1 below lists the number of passengers and crew who survived and died (the possible outcomes) in the sinking of the Titanic, categorized by status (first-class, second-class, third-class, and crew).

Titanic Table 1: Status and Survival

	First	Second	Third	Crew	
Survived	203	118	178	212	711
Died	122	167	528	673	1490
	325	285	706	885	2201

Source: British Wreck Commissioner's Inquiry Report

1. What is the probability a passenger survived?

$$P(\text{surv}) = \frac{n(\text{surv})}{n(\text{total})} = \frac{711}{2201} = \boxed{32.3\%}$$

2. What is the probability a passenger survived and was in second class?

Let A = passenger survived　　　　Let B = second class

Then $P(A \cap B)$ = the "intersection" of A and B, what A and B have in common.

$$P(A \cap B) = \frac{N(A \cap B)}{n(\text{total})} = \frac{118}{2201} = \boxed{5.4\%}$$

3. What is the probability a passenger survived given they were in second class?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{118}{\cancel{2201}}}{\frac{285}{2201}} = \frac{118}{\cancel{2201}} \cdot \frac{\cancel{2201}}{285} = \frac{118}{285}$$

$$= \boxed{41.4\%}$$

Definition of Conditional Probability

The **conditional probability of an event** B given an event A, written $P(B \mid A)$, is $\frac{P(A \cap B)}{P(A)}$.

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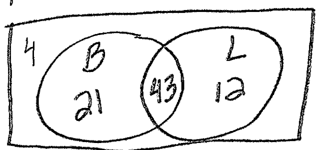

43 - Brkfst + Lunch
21 - Brkfst, not lunch
12 - Lunch, not brkfst

Example 1: Let B = a person eats a good breakfast; Let L = a person eats a good lunch. Suppose in a group of 80 people, 43 eat good breakfasts and good lunches, 21 eat a good breakfast but not a good lunch, 12 eat a good lunch but not a good breakfast, and the rest eat neither a good lunch nor a good breakfast.

a. Find $P(B \cap L)$ $\frac{N(B \cap L)}{P(\text{Total})} = \frac{43}{80} = \boxed{53.75\%}$

b. Find $P(L \mid B)$
Lunch given Breakfast

$$\frac{P(L \cap B)}{P(B)} = \frac{\frac{43}{80}}{\frac{21+43}{80}} \cdot \frac{80}{64} = \frac{43}{64} = \boxed{67.19\%}$$

c. Find $P(B \mid L)$
Breakfast given Lunch

$$\frac{P(B \cap L)}{P(L)} = \frac{\frac{43}{80}}{\frac{55}{80}} \cdot \frac{80}{55} = \frac{43}{53} = \boxed{78.18\%}$$

Example 2: An article in the Journal of the American Medical Association in 1997 reported that, when people go to their doctor's office with a sore throat and think they might have strep throat, 30% actually have strep throat. It noted that a current test for strep throat was 80% accurate if you have strep throat and 90% accurate if you do not. What is the probability that a person who receives a positive result from this test does not have the disease?

	Have Strep 30%	Do not have Strep 70%
Positive	80% $(.3)(.8) = 0.24$	10% $(.7)(.1) = 0.07$
Negative	20% $(.3)(.2) = 0.06$	90% $(.7)(.9) = 0.63$
	$= 100\%$	$= 100\%$

P (No strep given positive test) $= \boxed{22.6\%}$

$$\frac{P(\text{Both})}{P(\text{given})} = \frac{P(\text{No strep and } +)}{P(+)} = \frac{0.07}{0.24 + 0.07} = \frac{0.07}{0.31}$$

HM

Example 3: Suppose that 1 in 500 airline passengers carry some hazardous material on them when on a plane. Further suppose that an airport screening device accurately identifies 98% of people with hazardous materials that pass through it, and accurately identifies 99% of people without hazardous materials. If a person is identified by the machine as having hazardous materials, what is the probability that the person actually has these kinds of materials?

	HM $\frac{1}{500}$	No HM $\frac{499}{500}$
Identified with	98% $(\frac{1}{500})(.98) = .00196$	1% $(\frac{499}{500})(.01) = 0.00998$
Not identified with	2% $(\frac{1}{500})(.02) = .00004$	99% $(\frac{499}{500})(.99) = .98802$
	= 100%	= 100%

P(HM given Identified with HM)

$$\frac{P(\text{HM and ID'd with})}{P(\text{ID'd with})} = \frac{.00196}{.00196 + .00998}$$

$$= \frac{0.00196}{.01194}$$

$$= \boxed{16.4\%}$$