PC Trig Chapter 3 Review **Name**_____

1) State the amplitude and period of the graph of the following:

a) $y=5-\frac{3}{4}cos\left(\frac{1}{2}x\right)$

Amplitude: $3/4$

Period: $\frac{2\pi}{B}=\frac{2\pi}{\frac{1}{2}}\cdot\frac{2}{1}=4\pi$

b) $y=-2+\frac{3}{8}sin(2\pi x)$

Amplitude: $3/8$

Period: $\frac{2\pi}{B}=\frac{2\pi}{2\pi}=1$

c) $y=-5\,cot\left(\frac{2}{3}x+\frac{\pi}{3}\right)$

Amplitude: 5

Period: $\frac{\pi}{B}=\frac{\pi}{\frac{2}{3}}\cdot\frac{3}{2}=\frac{3\pi}{2}$

Phase Shift: $-\frac{C}{B}=\frac{-\frac{\pi}{3}\cdot\frac{3}{2}}{\frac{2}{3}}=\frac{-\pi}{2}$

d) $y=3\,tan\left(\frac{1}{2}x-\frac{\pi}{2}\right)$

Amplitude: 3

Period: $\frac{\pi}{\frac{1}{2}}\cdot\frac{2}{1}=2\pi$

Phase Shift: $-\frac{C}{B}=\frac{\frac{\pi}{2}\cdot\frac{2}{1}}{\frac{1}{2}}=\pi$

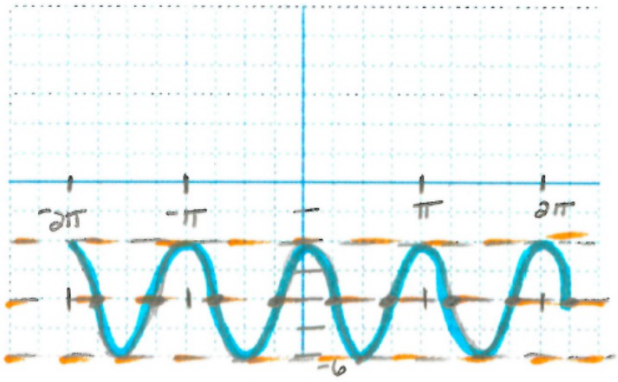
2) Sketch the graph of $y = -4 + 2\cos(2x)$ over at least one period. Clearly label the x and y-axis.

Amplitude: 2

Period: $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{-C}{B} = \frac{0}{2} = 0$

Vertical Shift: Down 4



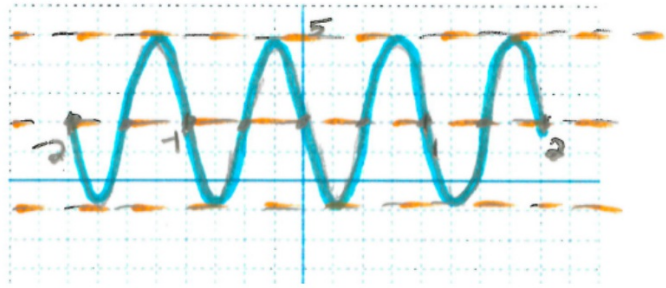
3) Sketch the graph of $y = 2 - 3\sin(2\pi x)$ over at least one period. Clearly label the x and y-axis.

Amplitude: 3 * FLIP *

Period: $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$

Phase Shift: $\frac{-C}{B} = \frac{-0}{2\pi} = 0$

Vertical Shift: up 2



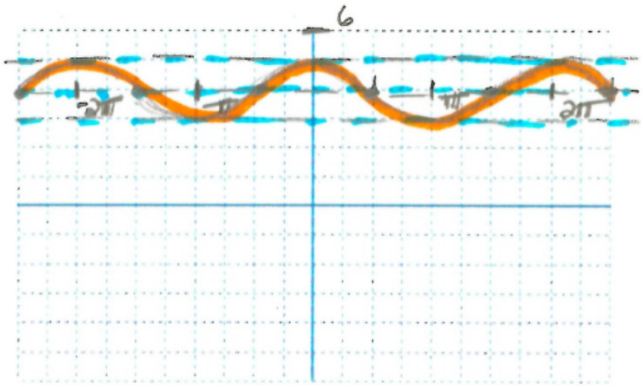
4) Sketch the graph of $y = 4 - \sin\left(x - \frac{\pi}{2}\right)$ over at least one period. Clearly label the x and y-axis.

Amplitude: 1 ★ FLIP ★

Period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

Phase Shift: $\frac{-C}{B} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$

Vertical Shift: Up 4



Find an equation of the form $y = k + A\cos Bx$ that produces the graph shown in the graphing calculator display in the following figure.

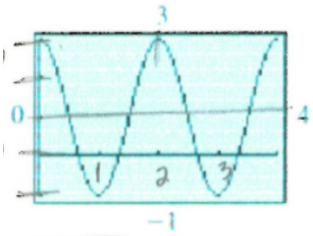


Figure for 33

$Amp = 2 \rightarrow A$

$P = \frac{2\pi}{B}$

$2 = \frac{2\pi}{B}$

$K = 1$

$B = \pi$

$y = 1 + 2\cos(\pi x)$

6)

Find the equation of the form $y = A\sin Bx$ whose graph is shown in the figure that follows.

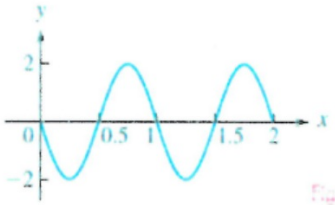


Figure for 34

~~AFLIPA~~

$Amp = 2 \rightarrow A$

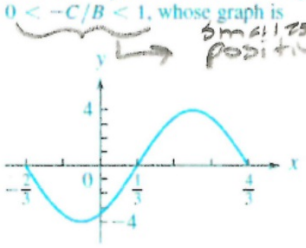
$P = \frac{2\pi}{B}$

$1 = \frac{2\pi}{B}$

$B = 2\pi$

$y = -2\sin(2\pi x)$

7) Find the equation of the form $y = A\sin(Bx + C)$, $0 < -C/B < 1$, whose graph is



small first positive phase shift

Amp = 4

Period = $\frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$

$2 = \frac{2\pi}{B}$

B = π

P.S. = $-\frac{C}{B}$

$\left(\frac{1}{3}\right)\pi = \left(\frac{-C}{\pi}\right)\pi$

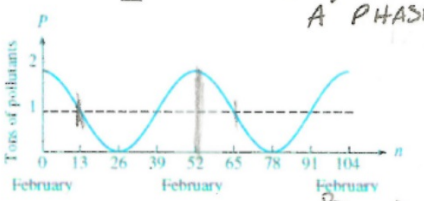
$-C = \frac{\pi}{3}$

$C = -\frac{\pi}{3}$

$y = 4\sin\left(\pi x - \frac{\pi}{3}\right)$

8) **Pollution** In a large city the amount of sulfur dioxide pollutant released into the atmosphere due to the burning of coal and oil for heating purposes varies seasonally. If measurements over a 2 yr period produced the graph shown, find an equation of the form $P = K + A\cos Bn$, $0 \le n \le 104$, where P is the number of tons of pollutants released into the atmosphere during the nth week after January 31. Can an equation of the form $P = k + A\sin Bn$ model the situation? If yes, find it. If no, explain why.

= NO, WOULD NEED A PHASE SHIFT.



K = 1 A = 1

$y = 1 + \cos\left(\frac{\pi}{52}x\right)$

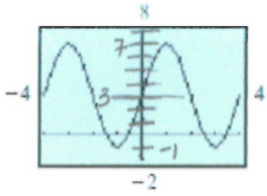
$P = B(52) = \left(\frac{2\pi}{B}\right)B$

$\frac{52B}{52} = \frac{2\pi}{52}$

$B = \frac{\pi}{26}$

3 4
↑ ↑

9) Find an equation of the form $y = k + A \sin Bx$ that produces the graph shown in the following graphing calculator display:



$P = \frac{2\pi}{B}$

$B(4) = \left(\frac{2\pi}{B}\right)B$

$\frac{4B}{4} = \frac{2\pi}{4}$

$B = \frac{\pi}{2}$

$y = 3 + 4\sin\left(\frac{\pi}{2}x\right)$

10) If the alternating voltage E in an electrical circuit has an amplitude of 12 V and a frequency of 40 Hz, and if E = 12 V when t = 0 sec, find an equation of the form $E = A\cos Bt$ that gives the voltage at any time t.

$A = 12$ $\quad P = \frac{1}{f} = \frac{1}{40}$

t | y
0 | 12

$y = 12\cos(80\pi x)$

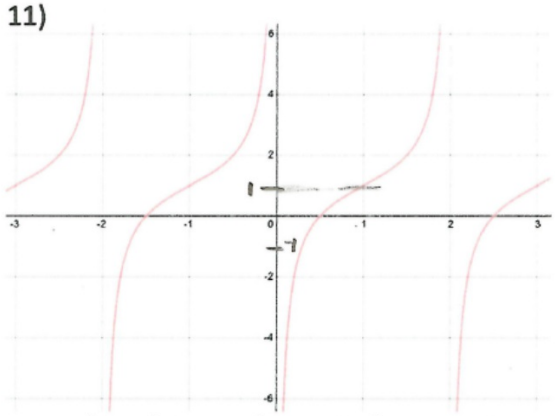
$P = \frac{2\pi}{B}$

$B\left(\frac{1}{40}\right) = \left(\frac{2\pi}{B}\right)B$

$40\left(\frac{B}{40}\right) = (2\pi)40$

$B = 80\pi$

11)



WINDOW
Xmin=-3
Xmax=3
Xscl=1/5
Ymin=-6
Ymax=6
Yscl=~~1/4~~ 1/2

$y = 1 + \tan\left(\frac{\pi}{2}x \pm \frac{\pi}{2}\right)$

A graph and its graphing window are pictured above. It has an equation in the form $y = k + \tan(Bx + C)$. Find k, B and C.

k: 1

B: $\frac{\pi}{2}$

$P = \frac{\pi}{B}$ $B(2) = \left(\frac{\pi}{B}\right)B$

$\frac{2B}{2} = \frac{\pi}{2}$ $B = \frac{\pi}{2}$

C: $-\frac{\pi}{2}$ -or- $\frac{\pi}{2}$

$P.S. = -\frac{C}{B}$ $\frac{\pi}{2}(1) = \left(-\frac{C}{\frac{\pi}{2}}\right)\frac{\pi}{2}$

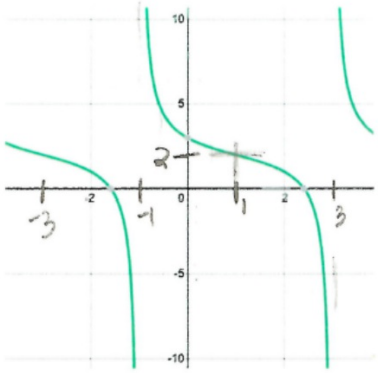
$-C = \frac{\pi}{2}$

$C = -\frac{\pi}{2}$

12)

WINDOW

12)



WINDOW
Xmin=-3.5
Xmax=3.5
Xscl=0.5
Ymin=-10
Ymax=10
Yscl=1

$y = 2 + \cot\left(\frac{\pi}{4}x + \frac{\pi}{4}\right)$

A graph and its graphing window are pictured above. It has an equation in the form $y = k + \cot(Bx + C)$. Find k, B and C.

k: 2

B: $\frac{\pi}{4}$

$P = \frac{\pi}{B}$ $\quad B(4) = \left(\frac{\pi}{B}\right)B$

$\frac{4B}{4} = \frac{\pi}{4}$ $\quad B = \frac{\pi}{4}$

C: $\frac{\pi}{4}$

$P.S. = \frac{-C}{B}$

$\frac{\pi}{4}(4) = \left(\frac{-C}{\frac{\pi}{4}}\right)\frac{\pi}{4}$

$-C = \frac{\pi}{4}$

$C = \frac{\pi}{4}$

3

In 13-16, match each equation with one of the following graphing utility displays. Explain how you made the choice relative to the period and phase shift (P.S.). (Show your work.)

13) $y = 3\sin\left(\pi x - \frac{\pi}{2}\right)$ Period: $\frac{2\pi}{\pi} = 2$ P.S.: $\frac{\frac{\pi}{2}}{\pi} \cdot \frac{1}{\pi} = \frac{1}{2}$ Graph: C

14) $y = 3\cos\left(4x + \frac{\pi}{2}\right)$ Period: $\frac{2\pi}{4} = \frac{\pi}{2}$ P.S.: $\frac{-\frac{\pi}{2}}{4} \cdot \frac{1}{4} = -\frac{\pi}{8}$ Graph: A

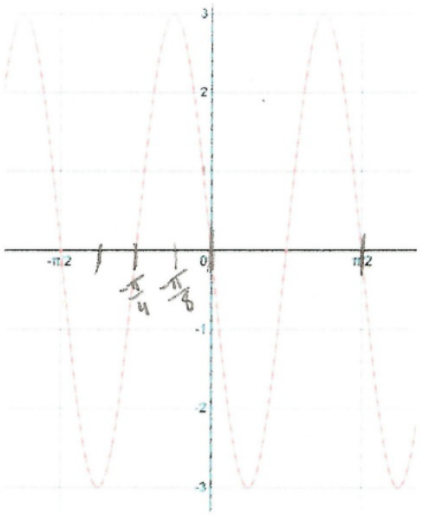
15) $y = 2\sin\left(\pi x + \frac{\pi}{2}\right)$ Period: $\frac{2\pi}{\pi} = 2$ P.S.: $\frac{-\frac{\pi}{2}}{\pi} \cdot \frac{1}{\pi} = -\frac{1}{2}$ Graph: D

16) $y = 3\cos\left(4x - \frac{\pi}{2}\right)$ Period: $\frac{2\pi}{4} = \frac{\pi}{2}$ P.S.: $\frac{\frac{\pi}{2}}{4} \cdot \frac{1}{4} = \frac{\pi}{8}$ Graph: B

Graph A $P = \frac{\pi}{2}$

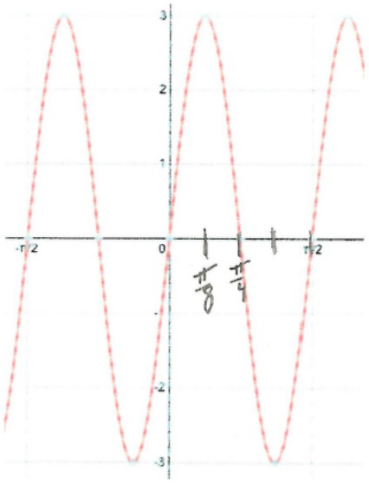
Graph B $P = \frac{\pi}{2}$

Graph A $P = \frac{\pi}{2}$



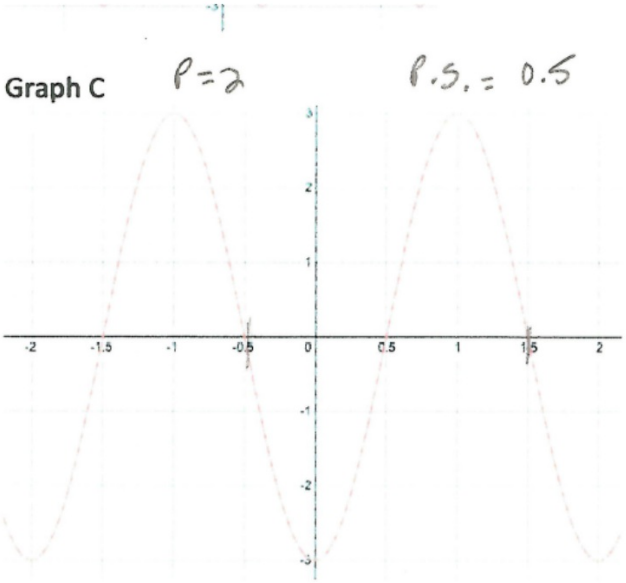
$P.S. = -\frac{\pi}{8}$

Graph B $P = \frac{\pi}{2}$

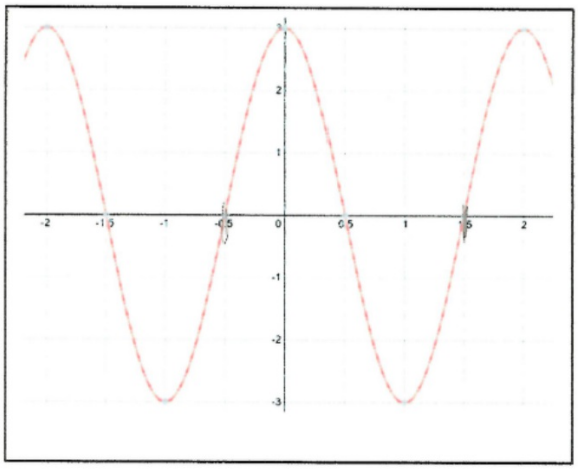


$P.S. = \frac{\pi}{8}$

Graph C $P = 2$ $P.S. = 0.5$



Graph D $P = 2$ $P.S. = -0.5$



17) Sketch the graph of $y = 2\csc\left(x + \frac{\pi}{4}\right)$ over at least one period. Clearly label the x and y-axis.

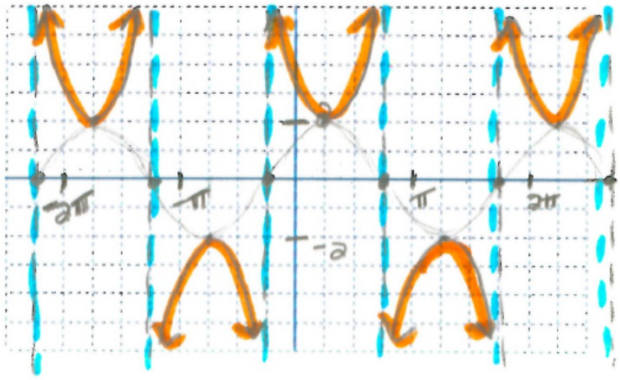
$\frac{1}{\sin}$

Amplitude: 2

Period: $\frac{2\pi}{1} = 2\pi$

Phase Shift: $\frac{-C}{B} = \frac{-\frac{\pi}{4}}{1} = -\frac{\pi}{4}$

Vertical Shift: None



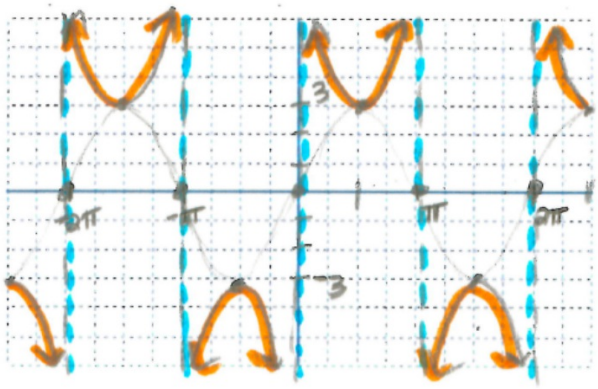
18) Sketch the graph of $y = 3\underbrace{sec}_{\frac{1}{\cos}}\left(x - \frac{\pi}{2}\right)$ over at least one period. Clearly label the x and y-axis.

Amplitude: 3

Period: $\frac{2\pi}{1} = 2\pi$

Phase Shift: $\frac{-c}{b} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$

Vertical Shift: None



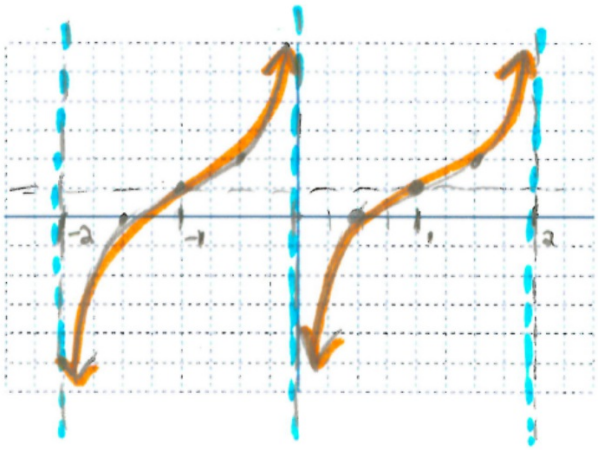
19) Sketch the graph of $y = 1 + \tan\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)$ over at least one period. Clearly label the x and y-axis.

Amplitude: 1

Period: $\frac{\pi}{B} = \frac{\pi}{\frac{\pi}{2}} \cdot \frac{2}{\pi} = 2$

Phase Shift: $-\frac{C}{B} = \frac{-\frac{\pi}{2} \cdot \frac{2}{\pi}}{\frac{\pi}{2}} = -1$

Vertical Shift: up 1



20) What is the amplitude, period, and phase shift of $y = -5\tan\left(\frac{2}{3}x - \frac{\pi}{5}\right)$.

Amplitude: 5 Period: $\frac{3\pi}{2}$ Phase Shift: $\frac{3\pi}{10}$

$\frac{\pi}{\frac{2}{3}} \cdot \frac{3}{2} = \frac{3\pi}{2}$ $\frac{-C}{B} = \frac{\frac{\pi}{5} \cdot \frac{3}{2}}{\frac{2}{3}} = \frac{3\pi}{10}$

21) What is the amplitude, period, and phase shift of $y = 8\tan(5x+6)$.

Amplitude: 8 Period: $\frac{\pi}{5}$ Phase Shift: $\frac{-6}{5}$

$\frac{\pi}{5}$

22)

Electrical Circuits If the voltage E in an electrical circuit has amplitude 18 and frequency 30 Hz, and $E = 18$ V when $t = 0$ sec, find an equation of the form $y = A \cos Bt$ that gives the voltage at any time t.

$A = 18$ $\quad P = \frac{1}{f} = \frac{1}{30}$ $\quad P = \frac{2\pi}{B}$

$B\left(\frac{1}{30}\right) = \left(\frac{2\pi}{B}\right)B$

$y = 18\cos(60\pi x)$ $\quad 30\left(\frac{B}{30}\right) = (2\pi)30$

$B = 60\pi$

23)

Spring–Mass System If the motion of a weight hung on a spring has an amplitude of 4 cm and a frequency of 8 Hz, and if its position when $t = 0$ sec is 4 cm below its position at rest (above the rest position is positive and below is negative), find an equation of the form $y = A \cos Bt$ that describes the motion at any time t (neglecting any damping forces such as air resistance and friction). Explain why an equation of the form $y = A \sin Bt$ cannot be used to model the motion. → An equation in this form would need to be at 0 cm at $t=0$.

$A = 4$ "$-$" due to being below at $t=0$

$P = \frac{1}{f} = \frac{1}{8}$

$B\left(\frac{1}{8}\right) = \left(\frac{2\pi}{B}\right)B$

$8\left(\frac{B}{8}\right) = (2\pi)8$

$B = 16\pi$

$y = -4\cos(16\pi t)$