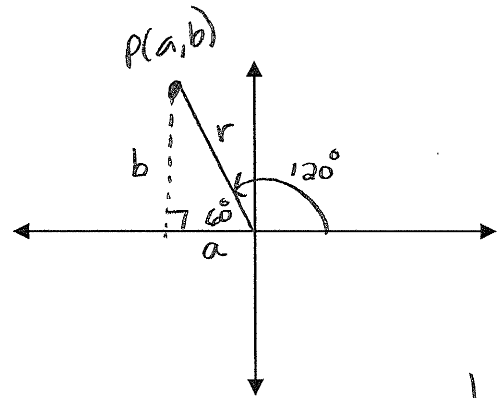
Trig - Section 2.3 - Reference Triangles and Angles

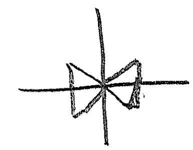
$r = \sqrt{a^2+b^2}$

For an arbitrary angle θ:

1) To form a reference triangle for θ, drop a perpendicular from a point P(a, b) on the terminal side of θ to the horizontal axis.



2) The reference angle α is the acute angle (always taken positive) between the terminal side of θ and the horizontal axis.

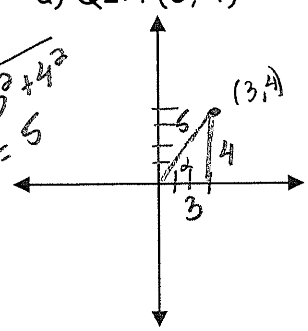


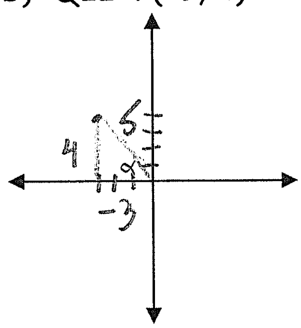
Trigonometric Functions with Angle Domains

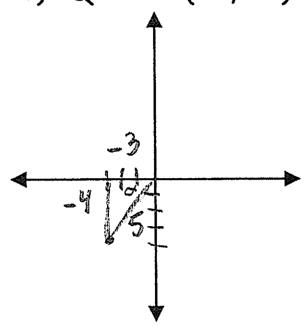
Note: P(a, b) is a point on the terminal side of θ, (a, b) ≠ (0, 0)

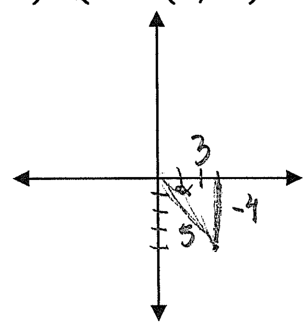
a) QI: P(3, 4) b) QII: P(-3, 4) c) QIII: P(-3, -4) d) QIV: P(3, -4)

$\sqrt{3^2+4^2} = 5$





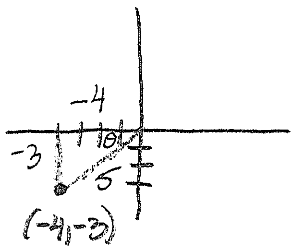




How would be determine the value of r?

sin θ $\frac{O}{H}$	QI $\frac{4}{5}$	QII $\frac{4}{5}$	csc θ	QI $\frac{5}{4}$	QII $\frac{5}{4}$
	QIII $\frac{-4}{5}$	QIV $\frac{-4}{5}$		QIII $\frac{5}{-4}$	QIV $\frac{5}{-4}$
cos θ $\frac{A}{H}$	QI $\frac{3}{5}$	QII $\frac{-3}{5}$	sec θ	QI $\frac{5}{3}$	QII $\frac{5}{-3}$
	QIII $\frac{-3}{5}$	QIV $\frac{3}{5}$		QIII $\frac{5}{-3}$	QIV $\frac{5}{3}$
tan θ $\frac{O}{A}$	QI $\frac{4}{3}$	QII $\frac{4}{-3}$	cot θ	QI $\frac{3}{4}$	QII $\frac{-3}{4}$
	QIII $\frac{-4}{-3} = \frac{4}{3}$	QIV $\frac{-4}{3}$		QIII $\frac{3}{4}$	QIV $\frac{3}{-4}$

1) Find the exact value of each of the six trigonometric functions for the angle θ with the terminal side containing P(-4, -3). (Like HW #3)

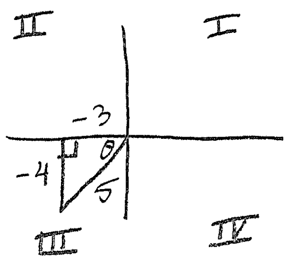


$\sin\theta = \frac{-3}{5}$ $\qquad$ $\csc\theta = \frac{5}{-3}$

$\cos\theta = \frac{-4}{5}$ $\qquad$ $\sec\theta = \frac{5}{-4}$

$\tan\theta = \frac{-3}{-4} = \frac{3}{4}$ $\qquad$ $\cot\theta = \frac{4}{3}$

2) Find the exact value of each of the other five trigonometric functions for the given angle θ , without finding θ, given that the terminal side of θ is in quadrant III and $\cos\theta = \frac{-3}{5}\cdot\frac{4}{4}$ (Like HW #6) $\qquad \sqrt{5^2-(-3)^2} = \sqrt{16} = 4$



$\sin\theta = \frac{-4}{5}$ $\qquad$ $\csc\theta = \frac{-5}{4}$

$\cos\theta = \frac{-3}{5}$ $\qquad$ $\sec\theta = \frac{-5}{3}$

$\tan\theta = \frac{-4}{-3} = \frac{4}{3}$ $\qquad$ $\cot\theta = \frac{3}{4}$

3) With a calculator, evaluate each to four significant digits. Pay attention to the MODE of your calculator! (Like HW #15-30)

D a) sin 286.38°
− 0.9594

R b) tan(3.427 rad)
0.2934

R c) cot 5.063 → No degree symbol, so radian
− 0.3657

D d) cos(-107° 35′)
− 0.3021

R e) sec(-4.799) → No degree symbol, so radian
11.56

D f) csc 192° 47′ 22″
− 4.517

4) The Department of Energy reports that wind-produced electricity will jump to close to 1% of the nation's total electrical output by the year 2010. A particular wind generator can generate alternating current given by the equation:

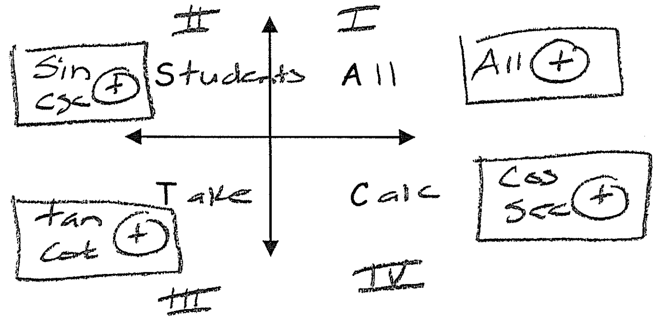
$I = 50\cos(120\pi t + 45\pi)$ ★ Radian Mode

where t is time in seconds and I is current in amperes. What is the current, I, (to 2 decimal places) when t = 1.09 sec? (Like HW #91)

$I = 50\cos(120\pi(1.09) + 45\pi)$

$I = 40.45$ amps

5) Which trig functions are positive in the:



1st Quadrant?
All 6 ratios

2nd Quadrant?
Sine
Cosecant

3rd Quadrant?
Tangent
Cotangent

4th Quadrant?
Cosine
Secant

6) In which quadrants must the terminal side of an angle θ lie in order for each of the following to be true?

a) $\overset{+}{sin\theta} > 0$

I, II

b) $\overset{+}{sec\theta} > 0$

I, IV

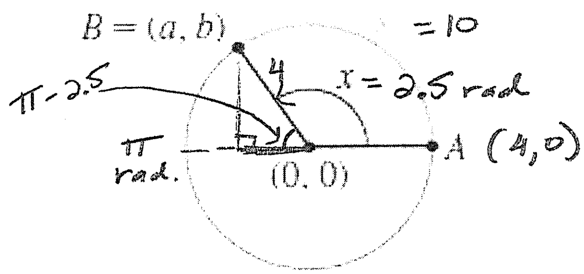
c) $\overset{-}{cos\theta} < 0$

II, III

d) $\overset{-}{csc\theta} < 0$

III, IV

7) In the figure, the coordinates of the center of the circle are (0,0). If the coordinates of A are (4, 0) and the arc length s is exactly 10 units, find:

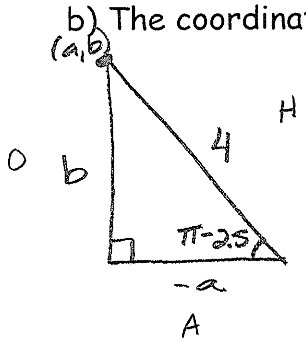


a) The exact radian measure of x .

$$X = \frac{S}{r} = \frac{10}{4} = \boxed{2.5 \text{ rad}}$$

b) The coordinates of B (to three significant digits).

★ Radian mode★



$$4\left(\sin(\pi - 2.5)\right) = \left(\frac{b}{4}\right)4$$

$$b = 2.39$$

$$4\left(\cos(\pi - 2.5)\right) = \left(\frac{a}{4}\right)4$$

$$a = 3.20 \rightarrow -3.20$$

$$\boxed{(a,b) \rightarrow (-3.20, 2.39)}$$