Trig Section 3-3 – Graphing $y = k + A\sin(Bx + C)$ and $y = k + A\cos(Bx + C)$

k: Vertical Shift
+ ↑
- ↓

$|A|$: amplitude
how far above and below horizontal axis

-A: reflected or flipped over horiz. axis

Period: $\dfrac{2\pi}{B}$

Phase Shift: (horizontal Shift): $\dfrac{-C}{B}$

*Shift **right** if $\dfrac{-C}{B} > 0$ — C must be "−"

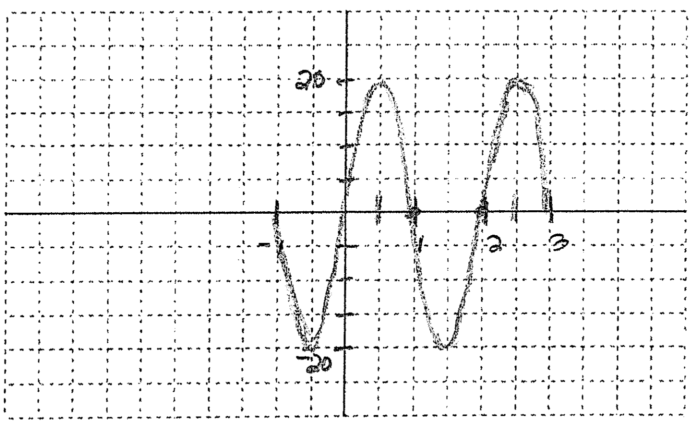
*Shift **left** if $\dfrac{-C}{B} < 0$ — C must be "+"

** In order to figure out where one cycle starts and ends, which will also tell you the period, solve the following 2 equations:

$Bx + C = 0$ and $Bx + C = 2\pi$

x = starting point of one cycle; x = ending point of one cycle

1) Graph $y = 20\cos\left(\pi x - \dfrac{\pi}{2}\right)$, $-1 \le x \le 3$



Amp = 20

$P = \dfrac{2\pi}{\pi} = 2$

Start

$\pi x - \dfrac{\pi}{2} = 0$

$\dfrac{\pi x}{\pi} = \dfrac{\frac{\pi}{2}}{\frac{\pi}{1}}$

$\boxed{x = \frac{1}{2}}$

End

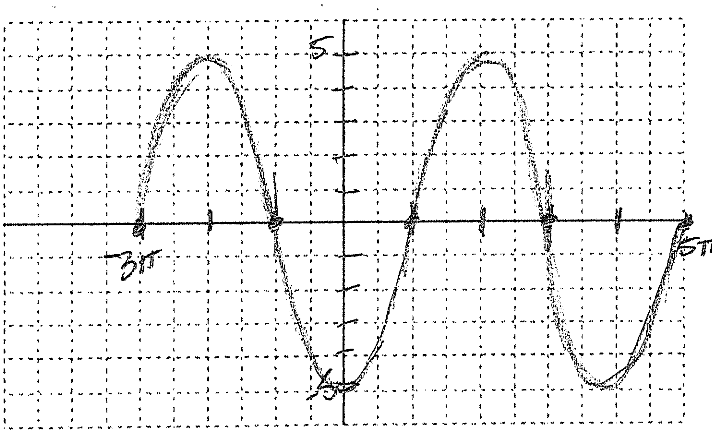
$\pi x - \dfrac{\pi}{2} = 2\pi$

$+\dfrac{\pi}{2} \quad +\dfrac{\pi}{2}$

$\dfrac{\pi x}{\pi} = \dfrac{\frac{5\pi}{2}}{\frac{\pi}{1}}$

$\boxed{x = \frac{5}{2} \text{ or } 2.5}$

2) Graph: $y = -5\sin\left(\dfrac{x}{2} + \dfrac{\pi}{2}\right)$ $-3\pi \le x \le 5\pi$



Amp = 5 ★ FLIP ★

$P = \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

Start

$\dfrac{x}{2} + \dfrac{\pi}{2} = 0$

$2\left(\dfrac{x}{2}\right) = \left(-\dfrac{\pi}{2}\right)2$

$\boxed{x = -\pi}$

End

$\dfrac{x}{2} + \dfrac{\pi}{2} = 2\pi$

$-\dfrac{\pi}{2} \quad -\dfrac{\pi}{2}$

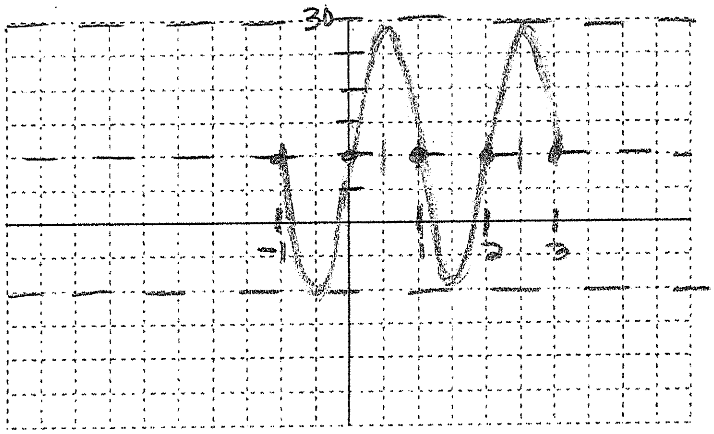
$2\left(\dfrac{x}{2}\right) = \left(\dfrac{3\pi}{2}\right)2$

$\boxed{x = 3\pi}$

3) Graph $y = 10 + 20\cos\left(\pi x - \frac{\pi}{2}\right)$, $-1 \le x \le 3$.

Amp = 20 ↑ 10

$P = \frac{2\pi}{\pi} = 2$



Start

$\pi x - \frac{\pi}{2} = 0$

$\frac{\pi x}{\pi} = \frac{\frac{\pi}{2}}{\pi}$

$\boxed{x = \frac{1}{2}}$

End

$\pi x - \frac{\pi}{2} = 2\pi$

$+\frac{\pi}{2} \quad +\frac{\pi}{2}$

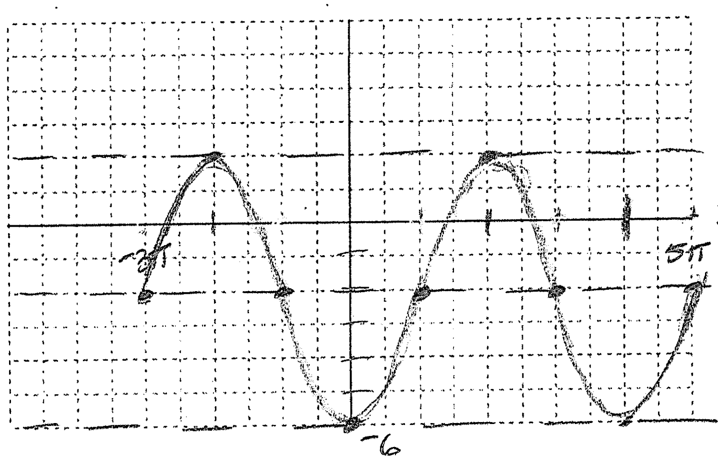
$\frac{\pi x}{\pi} = \frac{\frac{5}{2}\pi}{\pi}$

$\boxed{x = \frac{5}{2} \text{ or } 2.5}$

4) Graph $y = -2 - 4\sin\left(\frac{x}{2} + \frac{\pi}{2}\right)$, $-3\pi \le x \le 5\pi$.

Amp = 4 ✱ FLIP ↓ 2

$P = \frac{2\pi}{\frac{1}{2}} = 4\pi$



Start

$\frac{x}{2} + \frac{\pi}{2} = 0$

$2\left(\frac{x}{2}\right) = \left(-\frac{\pi}{2}\right)2$

$\boxed{x = -\pi}$

End

$\frac{x}{2} + \frac{\pi}{2} = 2\pi$

$-\frac{\pi}{2} \quad -\frac{\pi}{2}$

$2\left(\frac{x}{2}\right) = \left(\frac{3\pi}{2}\right)2$

$\boxed{x = 3\pi}$

5) Graph $y_1 = 4\sin x - 3\cos x$ on your calculator. Find an equation in the form $y_2 = A\sin(Bx + C)$ that has the same graph. Find A and B exactly and C to 3 decimal places. Amp = 5 ✱ Start (closest to origin) = 0.644

Period = End - Start

6.927 - 0.644

= 6.283 → 2π

Phase Shift = $\frac{-C}{B}$

$0.644 = \frac{-C}{1}$

$\boxed{C = -0.644}$

$\boxed{A = 5}$ $\boxed{2\pi = \frac{2\pi}{B} \quad B = 1}$ $\boxed{y = 5\sin(1x - 0.644)}$

6) Graph $y_1 = 3\sin x + 4\cos x$ on your calculator. Find an equation in the form $y_2 = A\sin(Bx + C)$ that has the same graph. Find A and B exactly and C to 3 decimal places. Amp = 5 ✱ Start = −0.927

Period = End - Start

5.356 − −0.927

= 6.283 → 2π

Phase Shift = $-\frac{C}{B}$

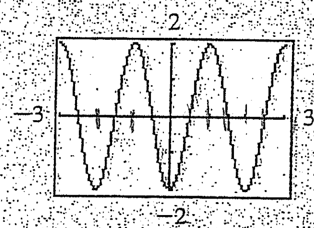
$-0.927 = \frac{-C}{1}$

$\boxed{C = 0.927}$

$\boxed{A = 5}$ $\boxed{2\pi = \frac{2\pi}{B} \quad B = 1}$ $\boxed{y = 5\sin(1x + 0.927)}$

EXPLORE/DISCUSS 1

Find an equation of the form $y = A\cos(Bx + C)$ that produces the graph in the following graphing calculator display (choose the smallest positive phase shift):



Is it possible for an equation of the form $y = A\sin(Bx + C)$ to produce the same graph? Explain. If it is possible, find the equation using the smallest positive phase shift. → YES

Amp = 2 Period = $2 = \frac{2\pi}{B}$ P.S. = $\frac{-C}{B}$

$B = \pi$

$\pi\left(1\right) = \left(\frac{-C}{\pi}\right)\pi$

$\pi = -C \qquad C = -\pi$

$\boxed{y = 2\cos(\pi x - \pi)}$

$\pi\left(\frac{1}{2}\right) = \left(-\frac{C}{\pi}\right)\pi$

$\frac{\pi}{2} = -C \qquad C = \frac{-\pi}{2}$

$\boxed{y = 2\sin\left(\pi x - \frac{\pi}{2}\right)}$

EXPLORE/DISCUSS 2

Explain why any function of the form $y = A\sin(Bx + C)$ can also be written in the form $y = A\cos(Bx + D)$ for an appropriate choice of D.

☆ Same amplitude and period, just different phase shifts.