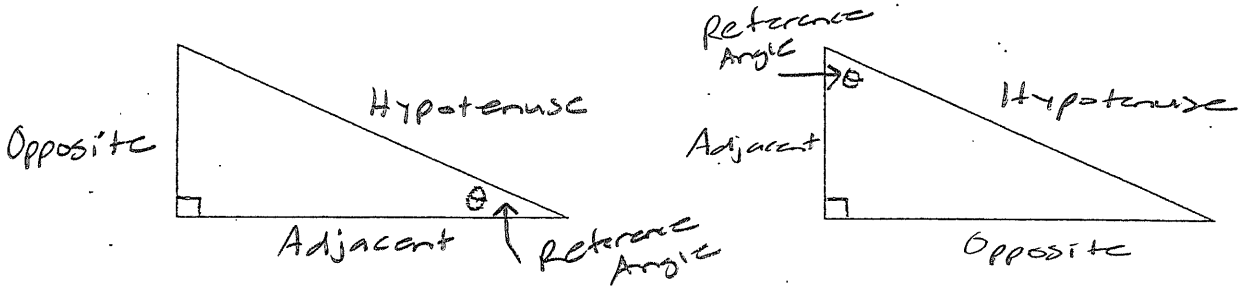
#3

TRIG – SECTION 1.3 – TRIG RATIOS & RIGHT TRIANGLES

Solving a Right Triangle: using the info given about the measures of a triangle's sides and angles to calculate the measures of the remaining sides and angles.

*use **Trigonometric Ratios** to solve **right** triangles



SOH	CAH	TOA
$\sin\theta = \frac{opposite}{hypotenuse}$	$\cos\theta = \frac{adjacent}{hypotenuse}$	$\tan\theta = \frac{opposite}{adjacent}$
$\csc\theta = \frac{hypotenuse}{opposite}$	$\sec\theta = \frac{hypotenuse}{adjacent}$	$\cot\theta = \frac{adjacent}{opposite}$

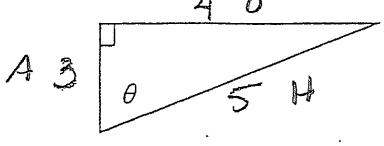
*Use **Reciprocal Relationships** to solve for cosecant (csc), secant (sec), and cotangent (cot).

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{O}{H}} = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{A}{H}} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{O}{A}} = \frac{\text{adjacent}}{\text{opposite}}$$

Ex : Find the 6 trig ratios for θ in the following triangle.



$\sin\theta = \frac{O}{H} = \frac{4}{5}$ $\quad \csc\theta = \frac{H}{O} = \frac{5}{4}$

$\cos\theta = \frac{A}{H} = \frac{3}{5}$ $\quad \sec\theta = \frac{H}{A} = \frac{5}{3}$

$\tan\theta = \frac{O}{A} = \frac{4}{3}$ $\quad \cot\theta = \frac{A}{O} = \frac{3}{4}$

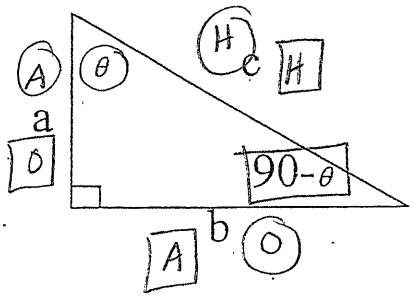
Complementary Angles and Cofunctions

*The 2 acute angles in a right triangle are always complementary..

Why? ↳ one 90° Angle, 3 ∠s add to 180°, so other 2 must add to 90°, two complementary

*Prefix co- in cosine, cosecant and cotangent refers to a complementary angle relationship. These 3 ratios are known as **Cofunctions**.

$\sin 30° = \cos(90° - 30°) = 0.5$



$\sin\theta = \frac{b}{c} = \cos(90 - \theta)$ ($\frac{O}{H}$ $\frac{A}{H}$)

$\tan\theta = \frac{b}{a} = \cot(90 - \theta)$ ($\frac{O}{A}$ $\frac{A}{O}$)

$\sec\theta = \frac{c}{a} = \csc(90 - \theta)$ ($\frac{H}{A}$ $\frac{H}{O}$)

DD: Decimal Degrees

DMS: Degrees, Minutes, Seconds

* each degree has 60 minutes and each minute has 60 seconds

Calculator Evaluation

*2 trigonometric modes: Radian and Degree

*We will work in **Degree** mode for now.

a) Radian b) Degree

Ex: Explore/Discuss 2, p. 25

Ex: Evaluate to 4 significant digits using a calculator :

a) sin 23.72° 0.4023 b) tan 54°37' 1.408 c) sec 49.31° 1.534

d) cot 12.86° 4.380 e) csc 77°53' 1.023

*Given the value of a ~~trig~~. ratio, we can compute the measure of an angle.

-We saw $\sin 23.72° = 0.4023$

$\theta = \arcsin 0.4023$ -or- $\theta = \sin^{-1} 0.4023$

$\theta = 23.72°$

NOTE: $\sin^{-1}.22$ is an inverse function and is **NOT the same as

$(\sin .22)^{-1} = \frac{1}{\sin .22} = 260.4$

$\sin^{-1}(.22) = 12.7 \neq 260.4$

$\sin^{-1}(.22) = (\sin .22)^{-1}$

Sec 1-3, p.3

Ex: Find each acute angle θ to the accuracy indicated:

a) $\tan\theta = 1.739$ (2 decimal places) 60.10

b) $\theta = \sin^{-1}0.2571$ (nearest 10") $14^\circ 53' 50''$ ★ Convert to DMS → 2nd APPS #4▶DMS

c) $\theta = \arccos 0.0367$ (nearest minute) $87^\circ 54'$

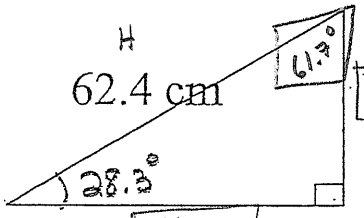
$87^\circ 53'$ 48.381" → more than half-way to whole minute, so round up to 54

Accuracy for Triangles (also in front cover of textbook)

Angle to Nearest	Sig. Dig. for side measure
1°	2
10' or 0.1°	3
1' or 0.01°	4
10" or 0.001°	5

Ex: Solve the following right triangles

a) $90^\circ - 28.3^\circ = 61.7^\circ$



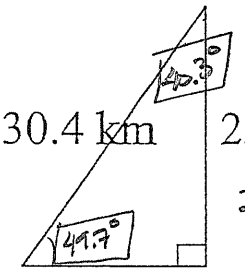
$62.4(\sin 28.3^\circ) = \left(\frac{b}{62.4}\right)62.4$

$b = 29.6\text{ cm}$

$62.4(\cos 28.3^\circ) = \left(\frac{a}{62.4}\right)62.4$

$a = 54.9$

b)



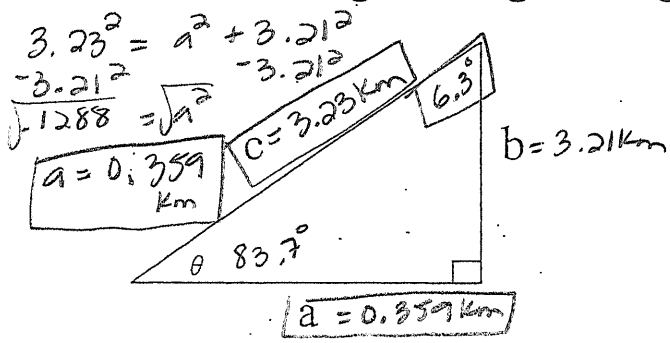
$\sin^{-1}\left(\frac{23.2}{30.4}\right) = 49.7^\circ$

$90 - 49.7 = 40.3^\circ$

$23.2(\tan 40.3^\circ) = \left(\frac{a}{23.2}\right)23.2$

$a = 19.7\text{ Km}$

Ex: Solve the right triangle using the given info.



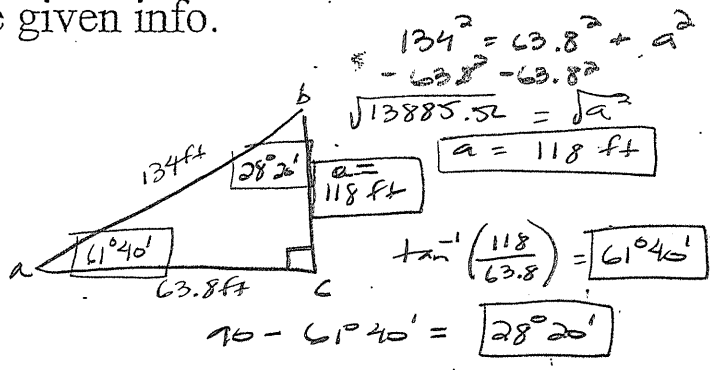
a) $\theta = 83.7^\circ$, b = 3.21km

$90 - 83.7 = 6.3^\circ$

$\frac{\sin 83.7^\circ}{1} = \frac{3.21}{c}$

$\frac{3.21}{\sin 83.7} = \frac{(\sin 83.7^\circ)c}{\sin 83.7}$

$c = 3.23\text{ Km}$



b) b = 63.8 ft, c = 134 ft (angles to nearest 10')

$\tan^{-1}\left(\frac{118}{63.8}\right) = 61^\circ 40'$

$90 - 61^\circ 40' = 28^\circ 20'$