TRIG – SECTION 2.1 – DEGREES AND RADIANS

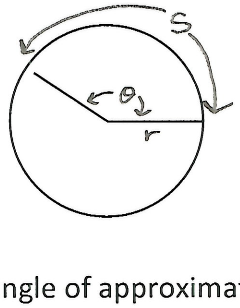
Radian Measure of Angles

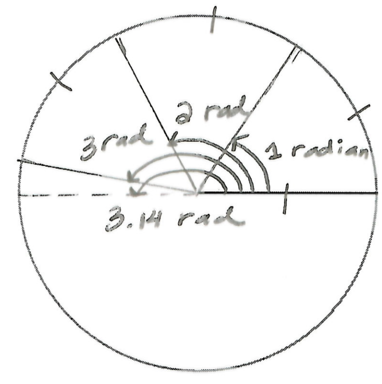
A central angle subtended by an arc of length equal to the radius of the circle is an angle of **radian measure 1**.

ARC LENGTH

$\theta = \frac{s}{r}$ radians (rad)

ANGLE MEASURE IN RADIANS — RADIUS





s and r must be same units

In the circle on the right draw an angle of approximately 1 radian, 2 radians and 3.141592654 radians.

Ex A: Find the radian measure of a central angle subtended by an arc 25 cm in a circle of radius 5 cm.

$\theta = \frac{s}{r} = \frac{25\text{ cm}}{5\text{ cm}} = \boxed{5 \text{ radians}}$

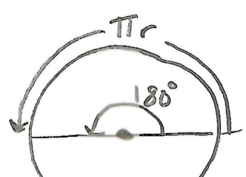
*Radian measure is a unitless number, so the word 'radian' is often omitted when we deal with the radian measure of angles.

What is the radian measure of an angle of 180°?

$\frac{C}{2} = \frac{2\pi r}{2} = \pi r$

- 180° subtends ½ the circumference of the circle
- $\frac{C}{2} = \frac{2\pi r}{2}$ so $r = \frac{C}{2\pi}$

$\frac{1}{2}C = \pi r$



$\theta = \frac{s}{r} = \frac{\pi r}{r} = \boxed{\pi}$

$\boxed{\begin{array}{l} \star\ 180° = \pi \text{ radians} \\ \star\ 360° = 2\pi \text{ radians} \end{array}}$

$\theta = \frac{s}{r} = \frac{2\pi r}{r} = \boxed{2\pi}$

**Unit Circle!

Degrees: 0, 90°, 180°, 270°, 360°

Radians: 0, $\pi/2$, π, $3\pi/2$, 2π

Radian-Degree Conversion

*Omit units in calculation until final answer

$\frac{\text{Degrees}}{180°} = \frac{\text{radians}}{\pi}$ – OR – $\frac{\text{Degrees}}{360°} = \frac{\text{radians}}{2\pi}$

Ex B: Find the degree measure of –1.5 rad in exact form and in decimal form to 4 decimal places.

$\frac{x}{180°} = \frac{-1.5}{\pi}$

$\frac{x\pi}{\pi} = \frac{-270}{\pi}$

$\boxed{x = -\frac{270°}{\pi}}$ – OR – $\boxed{x = -85.9437°}$

EXACT DECIMAL

Ex C: Find the radian measure of 44° in exact form and in decimal form to 4 decimal places.

$\frac{44°}{180°} = \frac{x}{\pi}$

$\frac{44\pi}{180} = \frac{x\,180°}{180}$

$x = \frac{44\pi}{180} = \boxed{\frac{11\pi}{45} \text{ rad}}$ – OR – $\boxed{0.7679 \text{ rad}}$

EXACT DECIMAL

*Can also use calc. to convert…
- put in mode that you are <u>converting to</u>
- let calc. know what you start with using ° or *r* in ANGLE menu

<u>Ex D</u>: Use a calculator to convert 44° to radians.

MODE → Radians , 44 , 2nd Apps , #1: ° , Enter $\boxed{0.7679 \text{ rad}}$

<u>Ex E:</u>

a) Find the degree measure of 1 rad in exact form and in decimal form to four decimal places.

$$\frac{1}{\pi} = \frac{x}{180}$$

$$\frac{180}{\pi} = \frac{x\pi}{\pi}$$

$$\boxed{x = \frac{180°}{\pi} = 57.2958°}$$

b) Find the radian measure of -120° in exact form and in decimal form to four decimal places.

$$\frac{-120°}{180°} = \frac{x}{\pi}$$

$$\frac{-120\pi}{180} = \frac{x\,180}{180}$$

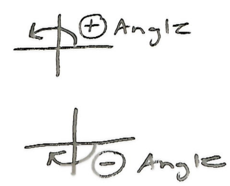
$$\boxed{x = -\frac{2}{3}\pi \text{ rad} = -2.0944 \text{ rad}}$$

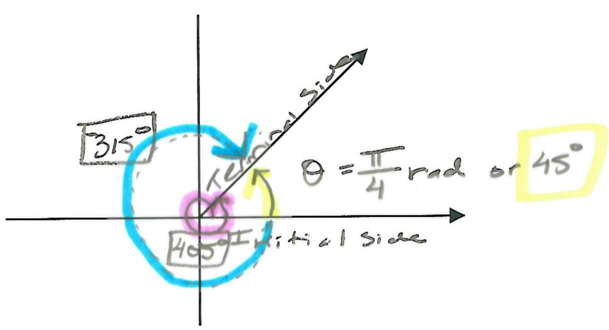
c) Use a calculator to perform the conversions in parts (A) and (B).

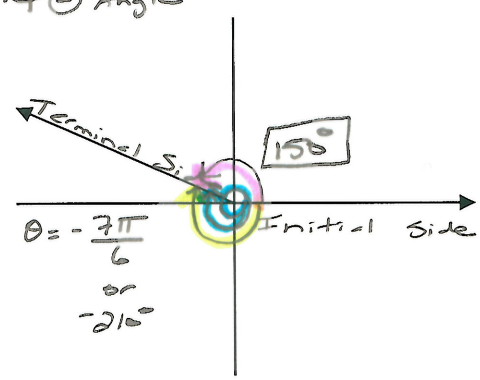
a) 1 rad → 57.2958°
- Degree mode
- 1, 2nd Apps, #3: r

b) -120° → -2.0944 rad
- Radian mode
- -120, 2nd Apps, #1: °

<u>Angle in Standard Position</u>
- vertex at origin
- initial side along positive x-axis
- positive angle rotates counterclockwise
- negative angle rotates clockwise







Coterminal Angles:

360° - 45° = 315° → $\boxed{315°}$

360° + 45° = 405° $\boxed{405°}$

Coterminal Angles:

360° - 210° = $\boxed{150°}$

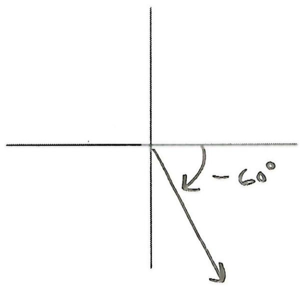
360° + 210° = 570° → $\boxed{-570°}$

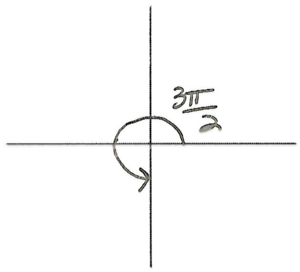
Recall <u>**Coterminal Angles**</u> share the same terminal sides when both angles are in standard position
- Degree measures of 2 coterminal angles differ by and integer multiple of 360°
- Radian measures differ by an integer multiple of 2π

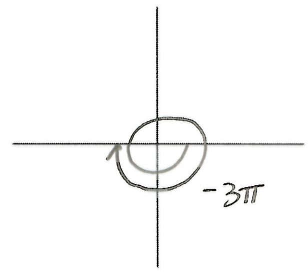
*Do ***Ex F*** and ***Ex G***, then find 2 angles, one positive, one negative, that are coterminal to the two given angles above.

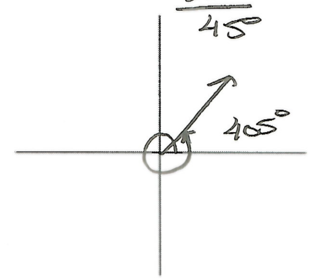
Ex F: Sketch the following angles in their standard positions.

a) -60^o b) $\frac{3\pi}{2} rad$ $\frac{3(180)}{2} = 270°$ c) $-3\pi \, rad$ d) 405^o $\begin{array}{r} -360 \\ \hline 45 \end{array}$



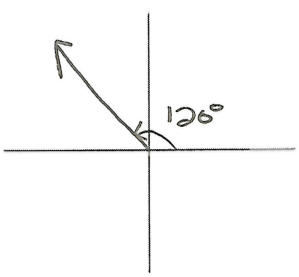


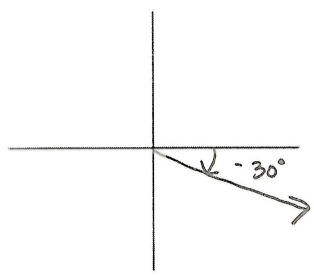


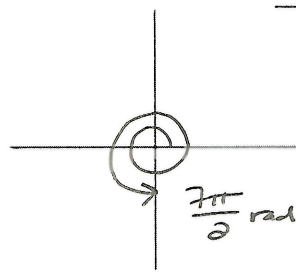


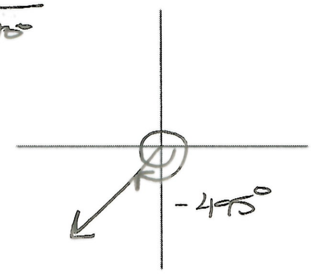
Ex G: Sketch the following angels in their standard positions.

a) 120^o b) $-\frac{\pi}{6} rad$ $\frac{-180}{6} = -30°$ c) $\frac{7\pi}{2} rad$ $\frac{7(180)}{2} = 630°$ $\begin{array}{r} -360 \\ \hline 270 \end{array}$ d) -495^o $\begin{array}{r} +360 \\ \hline -135 \end{array}$









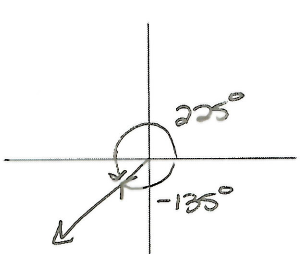
Ex H: Which of the following pairs are coterminal?

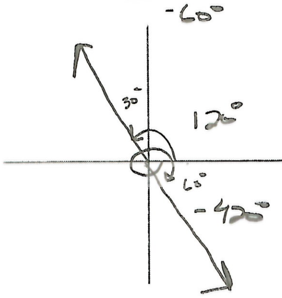
a) $\alpha = -135^o$ $\boxed{\text{Yes}}$
$\beta = 225^o$

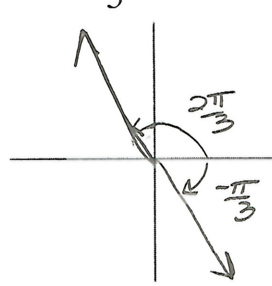
b) $\alpha = 120^o$ $\boxed{\text{No}}$
$\beta = -420^o$ $\begin{array}{r} +360 \\ \hline -60° \end{array}$

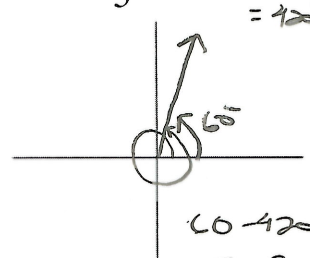
c) $\boxed{\text{No}}$
$\alpha = -\frac{\pi}{3} rad$ $\frac{-180}{3} = -60°$
$\beta = \frac{2\pi}{3} rad$ $\frac{2(180)}{3} = 120°$

d) $\boxed{\text{Yes}}$
$\alpha = \frac{\pi}{3} rad$ $\frac{180}{3} = 60°$
$\beta = \frac{7\pi}{3} rad$ $\frac{7(180)}{3} = 420°$









$60 - 420 = \frac{-360}{360} = \boxed{-1}$

$225 - -135 = \frac{360°}{360°} = \boxed{1}$ $120 - -420 = \frac{540°}{360°} = \boxed{1.5}$ $-60 - 120 = \frac{-180}{360} = \boxed{-\frac{1}{2}}$

Ex I: Which of the following pairs are coterminal?

a) $\boxed{\text{No}}$

$\alpha = 90^o$

$\beta = -90^o$

b) $\boxed{\text{Yes}}$

$\alpha = 750^o$

$\beta = 30^o$

c) $\boxed{\text{Yes}}$ $\quad \frac{-180}{6} = -30^\circ$

$\alpha = -\frac{\pi}{6} rad$

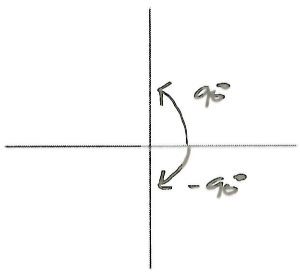
$\beta = -\frac{25\pi}{6} rad$

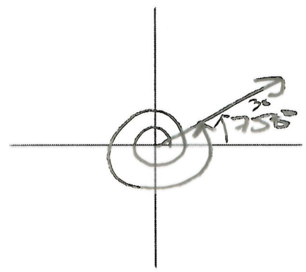
$\frac{-25(180)}{6} = -750^\circ$

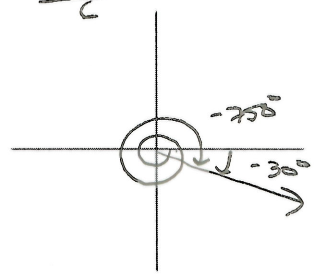
d) $\boxed{\text{No}}$ $\quad \frac{3(180)}{4} = 135^\circ$

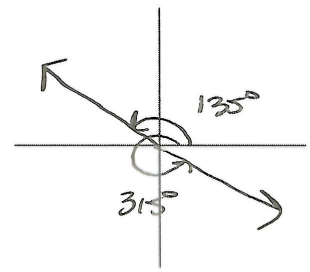
$\alpha = \frac{3\pi}{4} rad$

$\beta = \frac{7\pi}{4} rad$ $\quad \frac{7(180)}{4} = 315^\circ$









$90 - {}^-90 = \frac{180}{360} = \frac{1}{2}$

$750 - 30 = \frac{720}{360} = \boxed{2}$

$-30 - {}^-750 = \frac{720}{360} = \boxed{2}$

$135 - 315 = \frac{{}^-180}{360} = \boxed{{}^-1/2}$

Arc Length and Sector Area

Recall: $\frac{\text{central angle}}{360^\circ} = \frac{s}{2\pi r}$

s = arc length

Circumference = $2\pi r$

***Solve for s.**

In Radians: $\frac{\theta}{2\pi} = \frac{s}{2\pi r}$

$\frac{\theta(2\pi r)}{2\pi} = \frac{2\pi s}{2\pi}$

$\boxed{\theta r = s}$

Recall: $\frac{\text{central angle}}{360^\circ} = \frac{A}{\pi r^2}$

A = sector area

Area of Circle = πr^2

***Solve for A.**

In Radians: $\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$

$\frac{\theta \pi r^2}{2\pi} = \frac{2\pi A}{2\pi}$

$\boxed{\frac{\theta r^2}{2} = A}$

Ex J:
In a circle of radius 4.00 cm, find the arc length subtended by a central angle of:

a) 3.40 rad

$S = (3.40)(4.00)$

$\boxed{S = 13.6 \text{ cm}}$

b) 10.0°

$\frac{10}{360} = \frac{S}{2\pi(4)}$

$\boxed{S = 0.698 \text{ cm}}$

In a circle of radius 6.00 ft, find the arc length subtended by a central angle of:

c) 1.70 rad

$S = (1.70)(6.00)$

$\boxed{S = 10.2 \text{ ft}}$

d) 40.0°

$\frac{40}{360} = \frac{S}{2\pi(6)}$

$\boxed{S = 4.19 \text{ ft}}$

Ex K:
In a circle of radius 3 m., find the area (to three significant digits) of the circular sector with the central angle:

a) 0.4732 rad

$A = \frac{(0.4732)(3)^2}{2}$

$\boxed{A = 2.13 \text{ m}^2}$

b) 25°

$\frac{25°}{360°} = \frac{A}{\pi(3)^2}$

$\boxed{A = 1.96 \text{ m}^2}$

In a circle of radius 7 in., find the area (to four significant digits) of the circular sector with the central angle:

c) 0.1332 rad

$A = \frac{(0.1332)(7)^2}{2}$

$\boxed{A = 3.263 \text{ in}^2}$

d) 110°

$\frac{110°}{360°} = \frac{A}{\pi(7)^2}$

$\boxed{A = 47.04 \text{ in}^2}$