Homework Problems

In each of the following situations, X is a count. Does X have a binomial distribution? Explain.

1. You observe the gender of the next 40 children born in a hospital. X is the number of boys born.

2. You decide that you will have children until you have a boy or have a maximum of 5 children. X is the number of boys born.

3. I roll 10 dice. X is the number of 6's.

4. I roll 2 dice and add them. I continue to roll until I get a 7. X is the number of 7's I get.

5. It is estimated that 43% of people sleep regularly with a nightlight in their room. I take a sample of 35 people. X is the number of people who sleep regularly with a nightlight.

6. In a classroom of 15 students, 10 of them wear glasses or contacts. I choose 6 students. X is the number of them wearing glasses or contacts.

7. In an office building of 1,500 workers, 1,000 of them wear glasses or contacts. I choose 6 workers. X is the number of them wearing glasses or contacts.

8. Only 12% of people like pineapple pizza. I choose 25 people and give pizza with pineapple. Participants are allowed to take the pineapple off the pizza if they wish. X is the number of people who like the pizza.

9. An ice hockey player scores on 5.5% of his shots. In a particular game he gets 8 shots on goal. X is the number of goals he scores in the game.

10. When a paperback book is published, the probability that it has defects is .03%. A sample of 100 books are examined. X is the number of defective books in the sample.

Problems on Binomial Distributions

For each problem, be sure that the situation fits the criteria for binomial distributions. If so, answer the questions (show the formula) and then find the mean and standard deviation of the distribution.

1) 80% of the graduates of Northeast High who apply to Penn State University are admitted. Last year, there were 6 graduates from Northeast who applied to Penn State. What is the probability that

a) 4 were admitted

b) more than 4 were admitted

Mean of distribution: ____________

Standard deviation of distribution: ____________

2) Tires from the Apex Tire Corp. are traditionally 5% defective. A truck carries 10 tires, 8 in use and 2 spares. If 10 tires are chosen from Apex, what is the probability that not more than two defective tires are chosen.

Mean of distribution: ____________

Standard deviation of distribution: ____________

3) Studies indicate that in 70% of the families of Blue Bell, both the husband and wife work. If 7 families are randomly selected from Blue Bell, what is the probability that

a) exactly 4 of them work.

b) more than 4 work

Mean of distribution: ____________

Standard deviation of distribution: ____________

4) According to the National Institute of Health, 32% of all women will suffer a hip fracture because of osteoporosis by the age of 90. If 10 women aged 90 are selected at random, find the probability that

a) 2 or more of them suffer/will suffer a hip fracture

b) none of them suffer/will suffer a hip fracture

Mean of distribution: ____________

Standard deviation of distribution: ____________

5) According to the Internal Revenue Service, the chances of your tax return being audited are 3 in 100 if your income is $60,000 or less and 8 in 100 if your income is more than $60,000. 8 tax payers are chosen

a. earning less than $60,000. Find the probability that none will be audited.

b. earning more than $60,000. Find the probability that 4 or more are audited.

Mean of distribution: ____________

Standard deviation of distribution: ____________

6) According to FBI statistics, only 52% of all rape cases are reported to the police. If 10 rape cases are randomly selected, what is the probability that at least one is reported to the police?

Mean of distribution: ____________

Standard deviation of distribution: ____________

7) In a school, typically only $\frac{1}{10}$ of the student body returns surveys. 20 students are chosen randomly to receive a survey. What is the probability that

a) they get no surveys back.

b) they get more 4 or more surveys back.

Mean of distribution: ____________

Standard deviation of distribution: ____________

8) The probability that a driver making a gas purchase will pay by credit card is $\frac{3}{5}$. If 50 cars pull up to the station to buy gas, what is the probability that at least half of the drivers will pay by credit card?

Mean of distribution: ____________

Standard deviation of distribution: ____________

9) Light bulbs work out of the box 99.6% of the time. A contractor buys 50 bulbs. What is the probability that no more than two fail?

Mean of distribution: ____________ Standard deviation of distribution: ____________

10) in Lansdale, 44% of all fire alarms are false alarms. On a certain day, there were 12 fire alarms. Find the probability that

a) none is a false alarm

b) there are at least 2 false alarms.

Mean of distribution: ____________ Standard deviation of distribution: ____________

11) An ice hockey player scores on 5.5% of his shots. In a particular game he gets 8 shots on goal. Find the probability he scores 3 or more goals.

Mean of distribution: ____________ Standard deviation of distribution: ____________

12) An insurance company sells flood insurance to 1,000 customers. Statistics show that the probability of a flood in these homes in the coming year is 2.6%. What is the probability that they will have to pay a flood claim on

a) 20 or more homes

b) 40 or more homes

Mean of distribution: Standard deviation of distribution:

Geometric Probability Questions

1) A basketball player hits 76% of his free throws. He shoots until he misses. Complete the chart to find a) the probability that the first miss will occur on the *n*th free throw and b) the cumulative probability that the player will miss on or before the *n*th free throw.

n	1	2	3	4	5	6	7	8	9	10
$P(x=n)$										
Cumulative Probability										

Also, find the expected number of free throws to miss the first free throw and the probability that the player will shoot more than 10 free throws before missing

Finally, make a probability density histogram with the data above and a cumulative probability histogram.

2) In Buffalo, NY in January, the probability that it will snow on any day is 15%. Complete the chart to find a) the probability that the first day of snow in January will be on the given day and b) the cumulative probability that the first day of snow will occur that day or before.

Day (n)	Jan 1	Jan 2	Jan 3	Jan 4	Jan 5	Jan 6	Jan 7
$P(x=n)$							
Cumulative Probability							

Also, find the expected day of the first day of snow. And the probability that Buffalo will go more than a week before seeing snow.

Finally, make a probability density histogram with the data above and a cumulative probability histogram.

AP Statistics - Random Variables, Binomial, Geometric - Practice Test

Problems 1 – 9 deal with this situation: In a popular ice cream shop, people order ice cream cones with different number of scoops. If *X* represents the number of scoops ordered for randomly selected customers, then the following table gives the probability distribution of *X*.

X	1	2	3	4	5
P(*X*)	.21	.38	.27	.11	.03

1. Find $P(X \le 2)$

2. Find $P(X < 2 \text{ or } X > 4)$ - what does this mean in words?

3. Find the mean μ for this distribution. Show how you got your answer.

4. Find the variance for this distribution. Show how you got your answer.

5. Find the standard deviation for this distribution. Show how you got your answer.

6. Suppose the number of scoops of 25 randomly chosen customers is recorded. Call this number *Y*. Find the mean of *Y*.

7. Find the variance of *Y*.

8. Find the standard deviation of *Y*.

9. Suppose the business owners wanted to keep track of its costs. Ice cream scoops cost the business 62 cents and ice cream cones cost the business 8 cents. That would involve multiplying the number of ice cream scoops by $0.62 and adding $.08. Let Z = .62X + .08.

   a. Find μ_Z

   b. Find σ_Z

Problems 10 – 13 deal with this situation: According to statistics, 22.8% of people in the U.S. are vegetarians. 18 people are chosen at random.

10. If X is a random variable for this situation, define X in words. What kind of distribution does X have?

11. Find the following probabilities: (calculators allowed)

    a. that less than 4 are vegetarians.

    b. that at least half are vegetarians.

12. Find

    a. μ_X

    b. σ_X

13. For 18 people, find the probability that the number of people who are vegetarians is within 1.5 standard deviations of its mean.

Problems 14 – 16 deal with this situation: I take a multiple choice test with 6 questions, each having choice A, B, C, D, E. Let X be the number of questions I get right if I guess at each answer.

14. Complete the chart (3 decimal places)

X	0	1	2	3	4	5	6
P(X)							

15. Find μ_X

16. Show that σ_X can be calculated in two ways.

Problem 17 – 21 deal with this situation: At a gas station, only 19% of customers purchase premium gasoline.

17. What is the probability that 4 consecutive non-premium purchases will go by before a premium customer comes?

18. How many customers does the station expect to have before it gets a premium gas purchase?

19. What is the probability that more than 10 purchases are made before there is a premium gas purchase?

20. Construct a probability distribution table (out to n = 6) for the number of cars that will come into the station for a purchase to be premium.

X	1	2	3	4	5	6
P(X)						

21. Construct a probability histogram for the table you just constructed.

Homework Problems

In each of the following situations, X is a count. Does X have a binomial distribution? Explain.

1. You observe the gender of the next 40 children born in a hospital. X is the number of boys born.

$\boxed{\text{Yes. } n = 40, p \approx .5\text{. Children born are independent.}}$

2. You decide that you will have children until you have a boy or have a maximum of 5 children. X is the number of boys born.

$\boxed{\text{No. No value for } n.}$

3. I roll 10 dice. X is the number of 6's.

$\boxed{\text{No. There are more than two results (unless you consider a 6 is a success and not a 6 a failure).}}$

4. I roll 2 dice and add them. I continue to roll until I get a 7. X is the number of 7's I get.

$\boxed{\text{No. No value for } n.}$

5. It is estimated that 43% of people sleep regularly with a nightlight in their room. I take a sample of 35 people. X is the number of people who sleep regularly with a nightlight.

$\boxed{\text{Yes. } n = 35, p = .43\text{. Assume that the people chosen have independent sleeping habits.}}$

6. In a classroom of 15 students, 10 of them wear glasses or contacts. I choose 6 students. X is the number of them wearing glasses or contacts.

$\boxed{\text{No. Once you choose a person, the probability changes.}}$

7. In an office building of 1,500 workers, 1,000 of them wear glasses or contacts. I choose 6 workers. X is the number of them wearing glasses or contacts.

$\boxed{\text{Theoretically no. Once you choose a person, the probability changes, but so little that it acts like a binomial distribution.}}$

8. Only 12% of people like pineapple pizza. I choose 25 people and give pizza with pineapple. Participants are allowed to take the pineapple off the pizza if they wish. X is the number of people who like the pizza.

$\boxed{\text{No. The probability is not fixed.}}$

9. An ice hockey player scores on 5.5% of his shots. In a particular game he gets 8 shots on goal. X is the number of goals he scores in the game.

$\boxed{\text{Yes. } n = 8, p = .055\text{. Assume that his shooting any shot is independent.}}$

10. When a paperback book is published, the probability that it has defects is .03%. A sample of 100 books are examined. X is the number of defective books in the sample.

$\boxed{\text{Yes. } n = 100, p = .003\text{. Assume that defective books are independent.}}$

Problems on Binomial Distributions

For each problem, be sure that the situation fits the criteria for binomial distributions. If so, answer the questions (show the formula) and then find the mean and standard deviation of the distribution.

1) 80% of the graduates of Northeast High who apply to Penn State University are admitted. Last year, there were 6 graduates from Northeast who applied to Penn State. What is the probability that

a) 4 were admitted

$$\boxed{\binom{6}{4}(.8)^4(.2)^2 = \text{binompdf}(6,.8,4) = .246}$$

b) more than 4 were admitted

$$\boxed{\binom{6}{5}(.8)^5(.2)^1 + (.8)^6 = .655}$$

Mean of distribution: $\boxed{6(.8) = 4.8}$

Standard deviation of distribution: $\boxed{\sqrt{6(.8)(.2)} = .980}$

2) Tires from the Apex Tire Corp. are traditionally 5% defective. A truck carries 10 tires, 8 in use and 2 spares. If 10 tires are chosen from Apex, what is the probability that not more than two defective tires are chosen.

$$\boxed{(.05)^{10} + \binom{10}{1}(.05)^1(.95)^9 + \binom{10}{2}(.05)^2(.95)^8 = \text{binomcdf}(10,.05,2) = .988}$$

Mean of distribution: $\boxed{10(.05) = 0.5}$

Standard deviation of distribution: $\boxed{\sqrt{10(.05)(.95)} = .689}$

3) Studies indicate that in 70% of the families of Blue Bell, both the husband and wife work. If 7 families are randomly selected from Blue Bell, what is the probability that

a) exactly 4 of them work.

$$\boxed{\binom{7}{4}(.7)^4(.3)^3 = \text{binompdf}(7,.7,4) = .227}$$

b) more than 4 work

$$\boxed{\binom{7}{5}(.7)^5(.3)^2 + \binom{7}{6}(.7)^6(.3)^1 + (.7)^7 = .647}$$

Mean of distribution: $\boxed{7(.7) = 4.9}$

Standard deviation of distribution: $\boxed{\sqrt{7(.7)(.3)} = 1.212}$

4) According to the National Institute of Health, 32% of all women will suffer a hip fracture because of osteoporosis by the age of 90. If 10 women aged 90 are selected at random, find the probability that

a) 2 or more of them suffer/will suffer a hip fracture

$$\boxed{1 - (.68)^{10} - \binom{10}{1}(.32)^1(.68)^9 = .879}$$

b) none of them suffer/will suffer a hip fracture

$$\boxed{(.68)^{10} = .021}$$

Mean of distribution: $\boxed{10(.32) = 3.2}$

Standard deviation of distribution: $\boxed{\sqrt{10(.32)(.68)} = 1.475}$

5) According to the Internal Revenue Service, the chances of your tax return being audited are 3 in 100 if your income is $60,000 or less and 8 in 100 if your income is more than $60,000. 8 tax payers are chosen

a. earning less than $60,000. Find the probability that none will be audited.

$\boxed{(.97)^8 = .784}$

b. earning more than $60,000. Find the probability that 4 or more are audited.

$\boxed{1 - \text{binomcdf}(8,3,.08) = .002}$

Mean of distribution: $8(.03) = 24 \quad 8(.08) = .64$

Standard deviation of distribution: $\sqrt{8(.03)(.97)} = .482, \sqrt{8(.08)(.92)} = .767$

6) According to FBI statistics, only 52% of all rape cases are reported to the police. If 10 rape cases are randomly selected, what is the probability that at least one is reported to the police?

$$\boxed{1 - (.48)^{10} = .999}$$

Mean of distribution: $\boxed{10(.52) = 5.2}$ Standard deviation of distribution: $\boxed{\sqrt{10(.52)(.48)} = 1.580}$

7) In a school, typically only $\frac{1}{10}$ of the student body returns surveys. 20 students are chosen randomly to receive a survey. What is the probability that

a) they get no surveys back.

$\boxed{(.9)^{20} = .122}$

b) they get 4 or more surveys back.

$\boxed{1 - \text{binomialcdf}(20,.1,3) = .133}$

Mean of distribution: $\boxed{20(.1) = 2}$ Standard deviation of distribution: $\boxed{\sqrt{20(.1)(.9)} = 1.342}$

8) The probability that a driver making a gas purchase will pay by credit card is $\frac{3}{5}$. If 50 cars pull up to the station to buy gas, what is the probability that at least half of the drivers will pay by credit card?

$$\boxed{1 - \text{binomialcdf}(50,.6,24) = .943}$$

Mean of distribution: $\boxed{50(.6) = 3}$ Standard deviation of distribution: $\boxed{\sqrt{50(.6)(.4)} = 3.464}$

9) Light bulbs work out of the box 99.6% of the time. A contractor buys 50 bulbs. What is the probability that no more than two fail?

$$\boxed{(.996)^{50}+\binom{50}{1}(.004)^{1}(.996)^{49}+\binom{50}{2}(.004)^{2}(.996)^{48}=\text{binomcdf}(50,004,2)=.999}$$

Mean of distribution: $\boxed{50(.996)=49.}$ Standard deviation of distribution: $\boxed{\sqrt{50(.996)(.004)}=.446}$

10) In Lansdale, 44% of all fire alarms are false alarms. On a certain day, there were 12 fire alarms. Find the probability that

a) none is a false alarm

$\boxed{(.56)^{12}\approx 0}$

b) there are at least 2 false alarms.

$\boxed{1-\text{binomialcdf}(12,.44,1)=.990}$

Mean of distribution: $12(.44)=5.280$ Standard deviation of distribution: $\boxed{\sqrt{12(.44)(.56)}=1.720}$

11) An ice hockey player scores on 5.5% of his shots. In a particular game he gets 8 shots on goal. Find the probability he scores 3 or more goals.

$$\boxed{1-\text{binomialcdf}(8,.055,2)=.008}$$

Mean of distribution: $\boxed{8(.055)=.440}$ Standard deviation of distribution: $\boxed{\sqrt{8(.055)(.945)}=.645}$

12) An insurance company sells flood insurance to 1,000 customers. Statistics show that the probability of a flood in these homes in the coming year is 2.6%. What is the probability that they will have to pay a flood claim on

a) 20 or more homes

$\boxed{1-\text{binomialcdf}(1000,.026,19)=.906}$

b) 40 or more homes

$\boxed{1-\text{binomialcdf}(1000,.026,19)=.006}$

Mean of distribution: $\boxed{1000(.026)=26}$ Standard deviation of distribution: $\boxed{\sqrt{1000(.026)(.974)}=5.032}$

Geometric Probability Questions

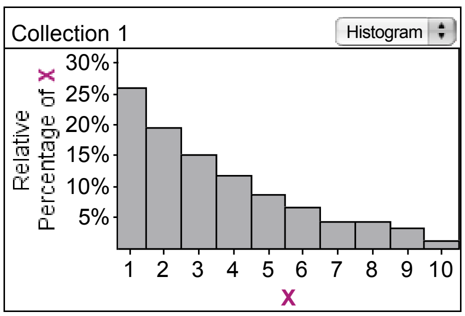
1) A basketball player hits 76% of his free throws. He shoots until he misses. Complete the chart to find a) the probability that the first miss will occur on the *n*th free throw and b) the cumulative probability that the player will miss on or before the *n*th free throw.

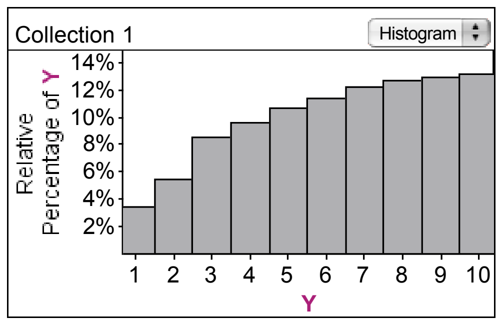
n	1	2	3	4	5	6	7	8	9	10
$P(x=n)$	.242	.182	.139	.105	.080	.061	.046	.035	.027	.020
Cumulative Probability	.240	.422	.561	.666	.746	.807	.854	.889	.915	.936

Also, find the expected number of free throws to miss the first free throw and the probability that the player will shoot more than 10 free throws before missing.

$$\frac{1}{.24} = 4.167 \qquad (.76)^{10} = .064$$

Finally, make a probability density histogram with the data above and a cumulative probability histogram.





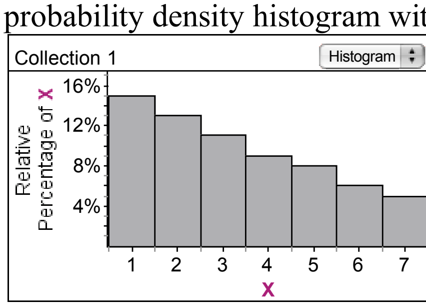
2) In Buffalo, NY in January, the probability that it will snow on any day is 15%. Complete the chart to find a) the probability that the first day of snow in January will be on the given day and b) the cumulative probability that the first day of snow will occur that day or before.

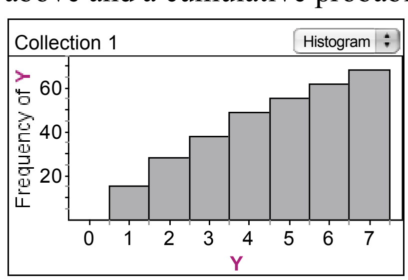
Day (n)	Jan 1	Jan 2	Jan 3	Jan 4	Jan 5	Jan 6	Jan 7
$P(x=n)$	.150	.128	.108	.092	.078	.067	.057
Cumulative Probability	.150	.278	.386	.478	.556	.623	.679

Also, find the expected day of the first day of snow. And the probability that Buffalo will go more than a week before seeing snow at the beginning of a year.

$$\frac{1}{.15} = 6.667 \ (\text{January } 7) \qquad (.85)^{7} = .321$$

Finally, make a probability density histogram with the data above and a cumulative probability histogram.





AP Statistics - Random Variables, Binomial, Geometric - Practice Test

Problems 1 – 9 deal with this situation: In a popular ice cream shop, people order ice cream cones with different number of scoops. If *X* represents the number of scoops ordered for randomly selected customers, then the following table gives the probability distribution of *X*.

X	1	2	3	4	5
P(*X*)	.21	.38	.27	.11	.03

1. Find $P(X \le 2)$
   $\boxed{.59}$

2. Find $P(X < 2 \text{ or } X > 4)$ - what does this mean in words?
   $\boxed{.24}$ - people either order very little or a lot of ice cream.

3. Find the mean μ for this distribution. Show how you got your answer.
   $\boxed{\mu = 1(.21) + 2(.38) + 3(.27) + 4(.11) + 5(.03) = 2.37}$

4. Find the variance for this distribution. Show how you got your answer.
   $\boxed{\sigma^2 = 1(.21 - 2.37)^2 + 2(.38 - 2.37)^2 + 3(.27 - 2.37)^2 + 4(.11 - 2.37)^2 + 5(.03 - 2.37)^2 = 1.053}$

5. Find the standard deviation for this distribution. Show how you got your answer.
   $\boxed{\sigma = \sqrt{1.053} = 1.026}$

6. Suppose the number of scoops of 25 randomly chosen customers is recorded. Call this number *Y*. Find the mean of *Y*.
   $\boxed{\mu_Y = \mu_{25X} = 25\mu_X = 59.25}$

7. Find the variance of *Y*.
   $\boxed{\sigma^2{}_{25X} = 25^2\sigma^2{}_X = 658.125}$

8. Find the standard deviation of *Y*.
   $\boxed{\sigma_{25X} = \sqrt{658.125} = 25.654}$

9. Suppose the business owners wanted to keep track of its costs. Ice cream scoops cost the business 62 cents and ice cream cones cost the business 8 cents. That would involve multiplying the number of ice cream scoops by \$0.62 and adding \$.08. Let Z = .62X + .08.

   a. Find μ_Z
   $\boxed{\mu_{.62X+.08} = .08 + .62\mu_X = 1.549}$

   b. Find σ_Z
   $\boxed{\sigma_{.62X+.08} = .62\sigma_X = .636}$

Problems 10 – 13 deal with this situation: According to statistics, 22.8% of people in the U.S. are vegetarians. 18 people are chosen at random.

10. If X is a random variable for this situation, define X in words. What kind of distribution does X have?
    X represents the number of people out of 18 (0 – 18) who are vegetarians. It is a binomial distribution.

11. Find the following probabilities: (calculators allowed)

    a. that less than 4 are vegetarians.
    $\boxed{P(\text{X} < 4) = .386}$
    Binomcdf(18,.228,3)

    b. that at least half are vegetarians.
    $\boxed{P(\text{X} \ge 9) = .011}$
    1 – Binomcdf(18,.228,8)

12. Find

    a. $\boxed{\mu_X = np = 18(.228) = 4.10}$

    b. $\boxed{\mu_X = \sqrt{np(1-p)} = \sqrt{18(.228)} = 1.78}$

13. For 18 people, find the probability that the number of people who are vegetarians is within 1.5 standard deviations of its mean.

You just found the standard deviation to be 1.78. 1.5(1.78) = 2.67. So you want the probability that X is between 4.10 - 2.67 and 4.10 + 2.67 meaning $P(1.43 < X < 6.77)$. Since X must be whole numbers, you want $P(X = 2 \text{ or } 3 \text{ or } 4 \text{ or } 5)$. These are binomial pdf's.

binompdf(18,.228,2) + binompdf(18,.228,3) + binompdf(18,.228,4) + binompdf(18,.228,5) = .$\boxed{730}$

Problems 14 – 16 deal with this situation: I take a multiple choice test with 6 questions, each having choice A, B, C, D, E. Let *X* be the number of questions I get right if I guess at each answer.

14. Complete the chart (3 decimal places)

X	0	1	2	3	4	5	6
P(*X*)	.262	.393	.246	.082	.015	.002	.000

15. Find μ_X

$\boxed{\mu_X = np = 6(.2) = 1.2}$

16. Show that σ_X can be calculated in two ways.

$\boxed{\sigma_X = \sqrt{(0-1.2)^2(.262) + (1-1.2)^2(.393) + \ldots} \text{ or } \sigma_X\sqrt{6(.25)(.75)}}$

Problem 17 – 21 deal with this situation: At a gas station, only 19% of customers purchase premium gasoline.

17. What is the probability that 4 consecutive non-premium purchases will go by before a premium customer comes?

$\boxed{(.81)^4(.19) = .082}$

18. How many customers does the station expect to have before it gets a premium gas purchase?

$\boxed{\frac{1}{.19} = 5.263}$

19. What is the probability that it takes more than 10 purchases to see the first premium sale?

$\boxed{.81^{10} = .122}$

20. Construct a probability distribution table (out to *n* = 6) for the number of cars that will come into the station for a purchase to be premium.

X	1	2	3	4	5	6
P(X)	.19	.154	.125	.101	.082	.066

21. Construct a probability histogram for the table you just constructed.