Question 1

Intent of Question

The primary goals of this question were to assess students' ability to (1) relate summary statistics to the shape of a distribution; (2) calculate and interpret a z-score; (3) make and justify a decision that involves comparing variables that are recorded on different scales.

Solution

Part (a):

No, it is not reasonable to believe that the distribution of 40-yard running times is approximately normal, because the minimum time is only 1.33 standard deviations below the mean $\left(z = \frac{4.4 - 4.6}{0.15} \approx -1.33\right)$. In a normal distribution, approximately 9.2 percent of the z-scores are below -1.33. However, there are no running times less than 4.4 seconds, which indicates that there are no running times with a z-score less than -1.33. Therefore, the distribution of 40-yard running times is not approximately normal.

Part (b):

The z-score for a player who can lift a weight of 370 pounds is $z = \frac{370 - 310}{25} = 2.4$. The z-score indicates that the amount of weight the player can lift is 2.4 standard deviations above the mean for all previous players in this position.

Part (c):

Because the two variables — time to run 40 yards and amount of weight lifted — are recorded on different scales, it is important not only to compare the players' values but also to take into account the standard deviations of the distributions of the variables. One reasonable way to do this is with z-scores.

The z-scores for the 40-yard running times are as follows:

$$\text{Player A: } z = \frac{4.42 - 4.60}{0.15} = -1.2$$
$$\text{Player B: } z = \frac{4.57 - 4.60}{0.15} = -0.2$$

The z-scores for the amount of weight lifted are as follows:

$$\text{Player A: } z = \frac{370 - 310}{25} = 2.4$$
$$\text{Player B: } z = \frac{375 - 310}{25} = 2.6$$

Question 1 (continued)

The z-scores indicate that both players are faster than average in the 40-yard running time and both are well above average in the amount of weight lifted. Player A is better in running time, and Player B is better in weight lifting. But the z-scores also indicate that the difference in their weight lifting (a difference of 0.2 standard deviation) is quite small compared with the difference in their running times (a difference of 1.0 standard deviation). Therefore, Player A is the better choice, because Player A is much faster than Player B and only slightly less strong.

Scoring

Parts (a), (b) and (c) are scored as essentially correct (E), partially correct (P) or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the answer is "no" *AND* the response provides a reasonable explanation, based on the relationship between the mean, standard deviation, and minimum value of a data set whose distribution can be approximated by a normal distribution.

Partially correct (P) if the answer is "no" but the explanation is weak.

Incorrect (I) if the answer is "no" without an explanation or with an unreasonable explanation, *OR* if the response concludes that it is reasonable to believe that the distribution is approximately normal.

Notes

- A reasonable explanation should describe a characteristic of a normal distribution that is substantially contradictory to the information given for the running time data so that the running time distribution cannot be reasonably approximated by a normal distribution.
- Plausible comments about the distribution of running times are considered extraneous.
- Incorrect comments about the distribution of running times can lower the score one level (that is, from E to P or from P to I), depending on the severity of the comment.

Part (b) is scored as follows:

Essentially correct (E) if the response calculates the z-score correctly *AND* provides a correct interpretation that includes direction.

Partially correct (P) if the response has only one of the two components (calculation and interpretation) correct.

Incorrect (I) if the response fails to meet the criteria for E or P.

Notes

- Calculating a probability from a normal distribution for the weights is considered extraneous and is not a sufficient interpretation of a z-score.
- Percentiles are extraneous and cannot be used to indicate direction from the mean, because the distribution cannot be determined from the information provided.
- Context is provided in the stem of problem and is not required for the response to be considered correct.

Question 1 (continued)

- Either the formula with correct symbols or with correct numerical values is needed in addition to the value 2.4 in the calculation of the z-score.
- A diagram can show direction from the mean, if the quantities are appropriately labeled.

Part (c) is scored as follows:

Essentially correct (E) if the response addresses the following three components:

1. Correct selection of Player A.
2. Numerical adjustments of the scales so that the players' values can be compared for *BOTH* variables: time to run 40 yards and amount of weight lifted.
3. Justification of the selection in component 1 by using the players' values on both variables with respect to the adjusted scales.

Partially correct (P) if the response has exactly two of the three components listed above.

Incorrect (I) if the response fails to meet the criteria for E or P.

Notes

- It is not necessary to calculate z-scores. For example, the following response is scored as essentially correct (E): "Players A and B are close in weight lifting, because the difference of 5 pounds is much less than 1 standard deviation (25 pounds), but much less close in running time because the difference is 0.15 seconds, which is exactly one standard deviation. Therefore, player A should be selected since he is considerably faster and almost as strong as player B."
- Component 3 is not satisfied by the statement, "Player A should be selected since the weights lifted are close and running times are less close," because the adjusted scales are not mentioned. Such a statement could apply to the original data, where the values are on different scales.
- The justification in component 3 must reference the adjusted scale for at least one variable *AND* at least be implied for the other variable.
- Normal probability calculations can be used in establishing the numerical scale adjustments for component 2 and for justifying the selection of the players in component 3. However, this results in a lowering of scores (that is, from E to P or from P to I) unless the student has concluded in part (a) that it was reasonable to believe that the distribution of running times was approximately normal.
- Conceptual miscalculation of z-scores or probabilities (for example, using the wrong mean, reversing the order of subtraction, or multiplying probabilities) results in the loss of credit for component 2, whereas minor arithmetic mistakes are overlooked.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

Question 1 (continued)

2 **Developing Response**

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 **Minimal Response**

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

STATISTICS
SECTION II
Part A
Questions 1-5
Spend about 65 minutes on this part of the exam.
Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. A professional sports team evaluates potential players for a certain position based on two main characteristics, speed and strength.

   (a) Speed is measured by the time required to run a distance of 40 yards, with smaller times indicating more desirable (faster) speeds. From previous speed data for all players in this position, the times to run 40 yards have a mean of 4.60 seconds and a standard deviation of 0.15 seconds, with a minimum time of 4.40 seconds, as shown in the table below.

	Mean	Standard Deviation	Minimum
Time to run 40 yards	4.60 seconds	0.15 seconds	4.40 seconds

Based on the relationship between the mean, standard deviation, and minimum time, is it reasonable to believe that the distribution of 40-yard running times is approximately normal? Explain.

$$z = \frac{x-\mu}{\sigma} = \frac{4.4-4.6}{.15} = \frac{-.2}{.15} = -1.3333$$

$$P(z<-1.3333) = .0912$$

No, it is not reasonable to believe that the distribution of 40-yard running times is approximately normal. If it were normal, one would expect about 9.12% of the performances to be better than 4.40 seconds, but in fact none of the performances are better than this time. Given that the data comes from all previous players of that position, the sample size should be large enough to produce a result with a p-value of .0912 if it were normal.

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(b) Strength is measured by the amount of weight lifted, with more weight indicating more desirable (greater) strength. From previous strength data for all players in this position, the amount of weight lifted has a mean of 310 pounds and a standard deviation of 25 pounds, as shown in the table below.

	Mean	Standard Deviation
Amount of weight lifted	310 pounds	25 pounds

Calculate and interpret the z-score for a player in this position who can lift a weight of 370 pounds.

$$z = \frac{x-\mu}{\sigma} = \frac{370-310}{25} = \frac{60}{25} = 2.4$$

The z-score for this player is 2.4. This means the weight he can lift is 2.4 standard deviations above the mean for players of that position.

(c) The characteristics of speed and strength are considered to be of equal importance to the team in selecting a player for the position. Based on the information about the means and standard deviations of the speed and strength data for all players and the measurements listed in the table below for Players A and B, which player should the team select if the team can only select one of the two players? Justify your answer.

	Player A	Player B
Time to run 40 yards	4.42 seconds	4.57 seconds
Amount of weight lifted	370 pounds	375 pounds

Player A $\quad \frac{4.42-4.6}{.15} = -1.2 \quad \frac{370-310}{25} = 2.4$

Player B $\quad \frac{4.57-4.6}{.15} = -.2 \quad \frac{375-310}{25} = 2.6$

The team should select player A. Player A is .2 standard deviation worse than player B in strength. However, Player A is 1 standard deviation better than Player B in speed. Therefore, Player A is better overall than player B because 1 > .2.

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STATISTICS
SECTION II
Part A
Questions 1-5

Spend about 65 minutes on this part of the exam.

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Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. A professional sports team evaluates potential players for a certain position based on two main characteristics, speed and strength.

   (a) Speed is measured by the time required to run a distance of 40 yards, with smaller times indicating more desirable (faster) speeds. From previous speed data for all players in this position, the times to run 40 yards have a mean of 4.60 seconds and a standard deviation of 0.15 seconds, with a minimum time of 4.40 seconds, as shown in the table below.

	Mean	Standard Deviation	Minimum
Time to run 40 yards	4.60 seconds	0.15 seconds	4.40 seconds

Based on the relationship between the mean, standard deviation, and minimum time, is it reasonable to believe that the distribution of 40-yard running times is approximately normal? Explain.

Based on the relationship between the mean, standard deviation and minimum time, it is not reasonable to believe that the distribution of 40-yard running times is approximately normal. This is due to the fact that, in order for such a distribution to be considered normal, 68% of the data must lie within 1 standard deviation of the mean, 95% of data within 2 standard deviations and 99.7% of data within 3 standard deviations. However, the minimum time recorded for this run is only 1.3 standard deviations from the mean. Therefore, the data cannot follow the rule outlined above.

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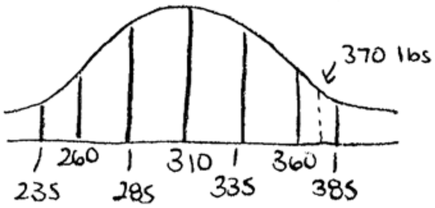
(b) Strength is measured by the amount of weight lifted, with more weight indicating more desirable (greater) strength. From previous strength data for all players in this position, the amount of weight lifted has a mean of 310 pounds and a standard deviation of 25 pounds, as shown in the table below.

	Mean	Standard Deviation
Amount of weight lifted	310 pounds	25 pounds

Calculate and interpret the z-score for a player in this position who can lift a weight of 370 pounds.

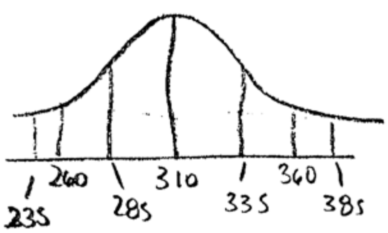


$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{370 - 310}{25} = 2.4$$

The z-score for a player in this position who can lift 370 lbs is 2.4.

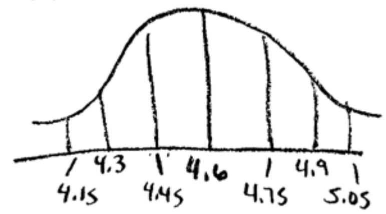
(c) The characteristics of speed and strength are considered to be of equal importance to the team in selecting a player for the position. Based on the information about the means and standard deviations of the speed and strength data for all players and the measurements listed in the table below for Players A and B, which player should the team select if the team can only select one of the two players? Justify your answer.

	Player A	Player B
Time to run 40 yards	4.42 seconds	4.57 seconds
Amount of weight lifted	370 pounds	375 pounds



$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{370 - 310}{25} = 2.4$$

$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{375 - 310}{25} = 2.6$$



$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{4.42 - 4.6}{.15} = -1.2$$

$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{4.57 - 4.6}{.15} = -.2$$

If only one player could be selected, the team should take Player A because, of the two, Player A is the overall better player. This is shown by the fact that the z-score of player A is one entire standard deviation from the mean lower than that of Player B in the speed category. Therefore, Player A is much faster. Also, Player A is only 0.2 standard deviations from the mean lower than Player B in the strength category. Therefore, overall Player A is much faster and only slightly weaker than Player B.

STATISTICS
SECTION II
Part A
Questions 1-5
Spend about 65 minutes on this part of the exam.
Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

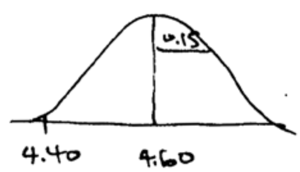
1. A professional sports team evaluates potential players for a certain position based on two main characteristics, speed and strength.

   (a) Speed is measured by the time required to run a distance of 40 yards, with smaller times indicating more desirable (faster) speeds. From previous speed data for all players in this position, the times to run 40 yards have a mean of 4.60 seconds and a standard deviation of 0.15 seconds, with a minimum time of 4.40 seconds, as shown in the table below.

	Mean	Standard Deviation	Minimum
Time to run 40 yards	4.60 seconds	0.15 seconds	4.40 seconds

Based on the relationship between the mean, standard deviation, and minimum time, is it reasonable to believe that the distribution of 40-yard running times is approximately normal? Explain.

If the distribution is approximately normal:



calculate the p-value of the minimum given → 4.40 seconds, and below.

$$z = \frac{4.40 - \mu}{\sigma}$$

$$= \frac{4.40 - 4.60}{0.15}$$

$$= -1.333$$

$P(z < -1.333) = 0.091$ or approximately 1%

∴ since the minimum value given is at one percentile if the distribution is normal with a mean of 4.60 s and a standard deviation of 0.15 seconds, it is reasonable to believe the distribution is approximately normal

close to min. of normal distribution

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(b) Strength is measured by the amount of weight lifted, with more weight indicating more desirable (greater) strength. From previous strength data for all players in this position, the amount of weight lifted has a mean of 310 pounds and a standard deviation of 25 pounds, as shown in the table below.

	Mean	Standard Deviation
Amount of weight lifted	310 pounds	25 pounds

Calculate and interpret the z-score for a player in this position who can lift a weight of 370 pounds.

$\mu = 310$ lb
$\sigma = 25$ lb

$$z = \frac{x - \mu}{\sigma} = \frac{370 - 310}{25} = 2.4$$

∴ the player can lift 2.4 standard deviations more than the mean.

$P(z < 2.4) = .992$

∴ it also means the player is in the 99th percentile and can lift more than approx. 99% of players in the position.

(c) The characteristics of speed and strength are considered to be of equal importance to the team in selecting a player for the position. Based on the information about the means and standard deviations of the speed and strength data for all players and the measurements listed in the table below for Players A and B, which player should the team select if the team can only select one of the two players? Justify your answer.

	Player A	Player B
Time to run 40 yards	4.42 seconds	4.57 seconds
Amount of weight lifted	370 pounds	375 pounds

Since speed & strength are of equal ~~[illegible]~~ importance and the numbers are very different (weight much larger than speed) we cannot add or subtract these number directly → use z-scores. (standardize)

	Player A	Player B
time	$z = \frac{4.42 - 4.60}{0.15} = -1.2$ ← use opposite sign when calculating total because faster is better →	$z = \frac{4.57 - 4.60}{0.15} = -0.2$
weight lifted	$z = \frac{370 - 310}{25} = 2.4$	$z = \frac{375 - 310}{25} = 2.6$
total	$2.4 + 1.2 = 3.6$ >	$2.6 + 0.2 = 2.8$

Since player A has the larger total z-score, the team should select that player.

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Question 1

Overview

The primary goals of this question were to assess students' ability to (1) relate summary statistics to the shape of a distribution; (2) calculate and interpret a z-score; (3) make and justify a decision that involves comparing variables that are recorded on different scales.

Sample: 1A
Score: 4

In part (a) a z-score of -1.3333 is calculated for the minimum running time of 4.40 seconds, and the probability of 0.0912 is calculated for z-scores less than -1.3333, using a normal distribution with mean 4.40 seconds and standard deviation 0.15 seconds. It is noted in the response that if the distribution of running times is normal, then 9.12 percent of the running times would be less than 4.40 seconds. Because there are no running times less than 4.40 seconds, the student concludes that a normal distribution cannot reasonably approximate the distribution of running times. Part (a) was scored as essentially correct. In part (b) the correct formula and the appropriate numerical values are used to calculate the correct z-score of 2.4. The correct interpretation of "2.4 standard deviations above the mean" is provided. Part (b) was scored as essentially correct. In part (c) the selection of Player A is justified by a comparison of the difference of the two z-scores for speed with the difference of the two z-scores for strength. Specifically, the response states, "Player A is .2 standard deviation worse than player B in strength"; "Player A is 1 standard deviation better than Player B in speed"; and "Player A is better overall than player B because $1 > .2$". Part (c) was scored as essentially correct. Because all three parts were scored as essentially correct, the response earned a score of 4.

Sample: 1B
Score: 3

In part (a) the response indicates that 68 percent, 95 percent, and 99.7 percent of a normal distribution are within 1, 2, and 3 standard deviations, respectively, of the mean, with the implication that the minimum value for an approximately normal distribution should be about 3 or more standard deviations below the mean. The minimum running time "is only 1.3 standard deviations from the mean," thus contradicting the belief that the running time data have an approximately normal distribution. Part (a) was scored as essentially correct. In part (b) an incorrect formula is given, but the appropriate numerical values are used in the formula, and the correct z-score value of 2.4 is computed. This was sufficient to receive credit for the calculation component. An interpretation of the z-score is not provided, so part (b) was scored as partially correct. In part (c) the conclusion that "Player A is much faster and only slightly weaker than Player B" is supported by describing their differences in running times and weight lifting in terms of standard deviations. Part (c) was scored as essentially correct. Because two parts were scored as essentially correct and one part was scored as partially correct, the response earned a score of 3.

Sample: 1C
Score: 2

In part (a) a correct z-score of -1.33 is calculated for the minimum running time. However, the area of a normal distribution below -1.33 is mistakenly reported to be 1 percent, which leads to an incorrect conclusion. Moreover, the student fails to consider the other parts of the distribution that must also be compared in order to approximate the running time distribution by a normal distribution. Part (a) was scored as incorrect. In part (b) the z-score is calculated and interpreted correctly. The calculation of a

Question 1 (continued)

normal probability for the z-score of 2.4 and the percentile interpretation were considered extraneous. Part (b) was scored as essentially correct. In part (c) the numerical adjustments of the variables, running times, and weight-lifting amounts are provided in terms of z-scores for both players. Moreover, it is indicated that the negative signs for running time z-scores must be changed to positive (because smaller times indicate faster speeds) for appropriate comparisons to the strength z-scores. The appropriate comparisons of the four z-scores are used in justifying the selection of Player A. Part (c) was scored as essentially correct. Because two parts were scored as essentially correct and one part was scored as incorrect, the response earned a score of 2.