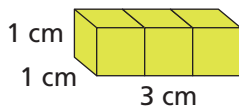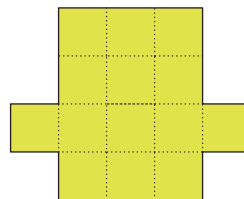# Investigation **2**

Designing Rectangular Boxes

Finding the right box for a product requires thought and planning. A company must consider how much the box can hold as well as the amount and the cost of the material needed to make the box.

The amount that a box can hold depends on its volume. The **volume** of a box is the number of unit cubes that it would take to fill the box. The amount of material needed to make or to cover a box depends on its surface area. The **surface area** of a box is the total area of all of its faces.

The box shown below has dimensions of 1 centimeter by 3 centimeters by 1 centimeter. It would take three 1-centimeter cubes to fill this box, so the box has a volume of 3 cubic centimeters. Because the net for the box takes fourteen 1-centimeter grid squares to make the box, the box has a surface area of 14 square centimeters.

1 cm
1 cm
3 cm

volume = 3 cubic centimeters

surface area = 14 square centimeters

In this investigation, you will explore the possible surface areas for a rectangular box that holds a given volume.

2.1 Packaging Blocks

ATC Toy Company is planning to market a set of children's alphabet blocks. Each block is a cube with 1-inch edges, so each block has a volume of 1 cubic inch.

Problem 2.1 Finding Surface Area

The company wants to arrange 24 blocks in the shape of a rectangular prism and then package them in a box that exactly fits the prism.

A. Find all the ways 24 cubes can be arranged into a rectangular prism. Make a sketch of each arrangement. Record the dimensions and surface area. It may help to organize your findings into a table like the one below:

Possible Arrangements of 24 Cubes

Length	Width	Height	Volume	Surface Area	Sketch
■	■	■	■	■	■
■	■	■	■	■	■
■	■	■	■	■	■

B. Which of your arrangements requires the box made with the least material? Which requires the box made with the most material?

C. Which arrangement would you recommend to ATC Toy Company? Explain why.

D. Why do you think the company makes 24 alphabet blocks rather than 26?

ACE Homework starts on page 24.

2.2 Saving Trees

You discovered that 24 blocks can be packaged in different ways that use varying amounts of packaging material. By using less material, a company can save money, reduce waste, and conserve natural resources.

Which rectangular arrangement of cubes uses the least amount of packaging material?

Problem 2.2 Finding the Least Surface Area

A. Explore the possible arrangements of each of the following numbers of cubes. Find the arrangement that requires the least amount of packaging material.

    1. 8 cubes   **2.** 27 cubes   **3.** 12 cubes

B. 1. Make a conjecture about the rectangular arrangement of cubes that requires the least packaging material.

    2. Does your conjecture work for 30 cubes? Does it work for 64 cubes? If not, change your conjecture so it works for any number of cubes. When you have a conjecture that you think is correct, give reasons why you think your conjecture is valid.

C. Describe a strategy for finding the total surface area of a closed box.

ACE **Homework starts on page 24.**

Area is expressed in square units, such as square inches or square centimeters. You can abbreviate square units by writing the abbreviation for the unit followed by a raised 2. For example, an abbreviation for square inches is in.^{2}.

Volume is expressed in cubic units. You can abbreviate cubic units by writing the abbreviation for the unit followed by a raised 3. For example, an abbreviation for cubic centimeters is cm^3.

Getting Ready for Problem 2.3

One seventh-grade student, Bernie, wonders if he can compare volumes without having to calculate them exactly. He figures that volume measures the contents of a container. He fills the prism on the left with rice and then pours the rice into the one on the right.

- How can you decide if there is enough rice or too much rice to fill the prism on the right?

4 cm

3 cm   3 cm

7 cm

5 cm   1 cm
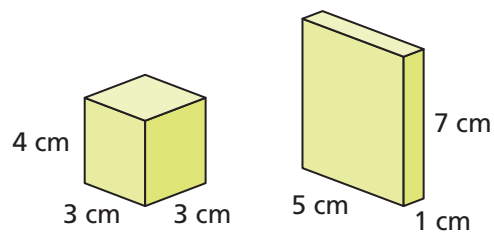
2.3 Filling Rectangular Boxes

A company may have boxes custom-made to package its products. However, a company may also buy ready-made boxes. The Save-a-Tree packaging company sells ready-made boxes in several sizes.

Box W

4 in.

3 in.   2 in.

Box X

3 in.

3 in.   3 in.

Box Y

2 in.

8 in.   5 in.

Box Z

10 in.

2 in.   4 in.
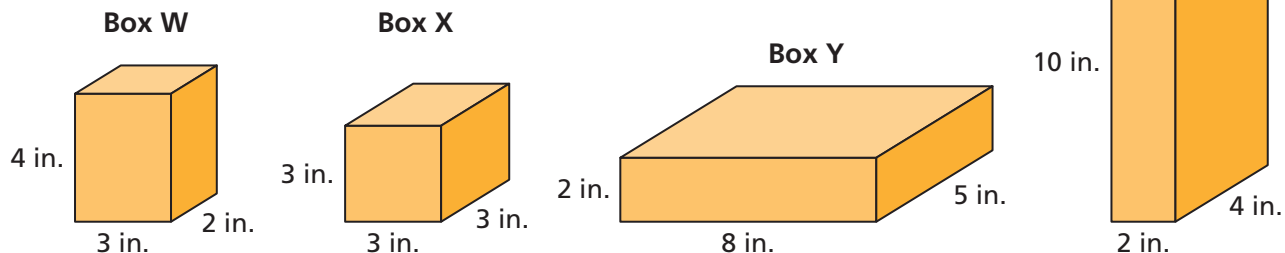
ATC Toy Company is considering using Save-a-Tree's Box Z to ship alphabet blocks. Each block is a 1-inch cube. ATC needs to know how many blocks will fit into Box Z and the surface area of the box.

A. The number of unit cubes that fit in a box is the volume of the box.

   1. How many cubes will fit in a single layer at the bottom of this box?

   2. How many identical layers can be stacked in this box?

   3. What is the total number of cubes that can be packed in this box?

   4. Consider the number of cubes in each layer, the number of layers, the volume, and the dimensions of the box. What connections do you see among these measurements?

10 in.

4 in.

2 in.
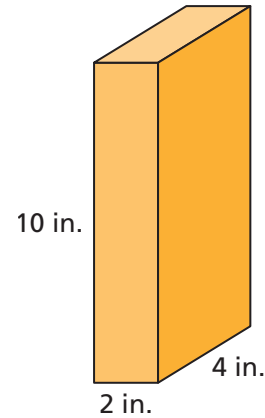
B. Find the surface area of Box Z.

C. Suppose Box Z is put down on its side so its base is 4 inches by 10 inches and its height is 2 inches. Does this affect the volume of the box? Does this affect the surface area? Explain your reasoning.

2 in.

4 in.

10 in.
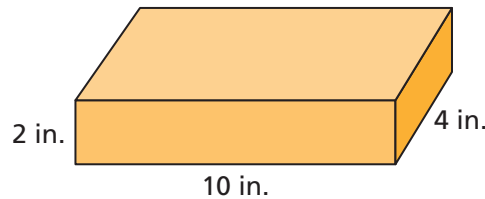
D. Apply your strategies for finding volume and surface area to Boxes W, X, and Y.