Investigation 3

Prisms and Cylinders

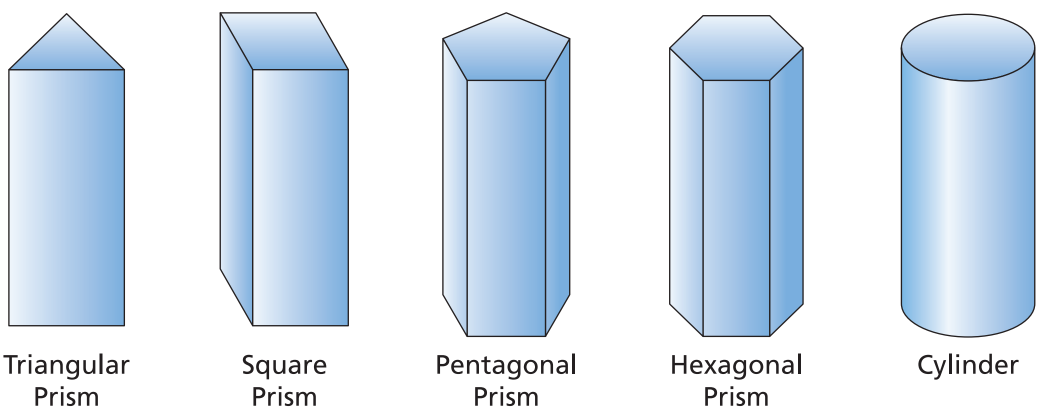
In Investigation 2, you found the volume of rectangular prisms by filling the prism with cubes. The number of cubes in the bottom layer is the same as the area of the rectangular base and the number of layers is the height. To find the volume, you multiply the area of the base ($\ell \times w$) times its height h, so that $V = \ell wh$.

A **prism** is a three-dimensional shape with a top and a base that are congruent polygons, and *lateral* (side) faces that are parallelograms. Each prism is named for the shape of its base. The boxes we have seen so far in this unit are rectangular prisms. A triangular prism has a triangular base.

A **cylinder** is a three-dimensional shape with a top and base that are congruent circles.

The prisms and cylinder below all have the same height.

Suppose you filled the triangular prism with rice and poured the rice into each of the other cylinders. How do you think the volumes would compare? What about the surface areas?

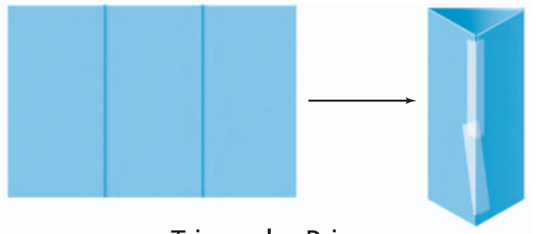


3.1 Filling Fancy Boxes

In this problem you will explore prisms with bases that are not rectangles. You will start by making models of prisms.

Directions for Making Paper Prisms (These paper models are open at the top and bottom.)

- Start with four identical sheets of paper.
- Use the shorter dimension as the height for each prism.
- Make a *triangular* prism by marking and folding one of the sheets of paper into three congruent rectangles. Tape the paper into the shape of a triangular prism.



Triangular Prism

- Make a *square* prism by marking, folding, and taping a sheet of paper into four congruent rectangles.
- Make a *pentagonal* prism by marking, folding, and taping a sheet of paper into five congruent rectangles.
- Make a *hexagonal* prism by marking, folding, and taping a sheet of paper into six congruent rectangles.

Problem 3.1 Finding the Volumes of Other Prisms

A. In your group, follow the directions above. (Keep your models for Problem 3.2.)

B. How do the volumes of the prisms compare as the number of faces in the prisms increases? Does the volume remain the same? Explain.

C. Consider the number of cubes you need to cover the base as one layer. Next, think about the total number of layers of cubes needed to fill the prism. Does this seem like a reasonable method for computing the volume of each prism?

D. Suppose that each of your paper prisms has a top and a bottom. As the number of faces of a prism (with the same height) increases, what happens to the surface area of the prisms? Are the surface areas of the prisms the same? Explain your reasoning.

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3.2 Filling Cylinders

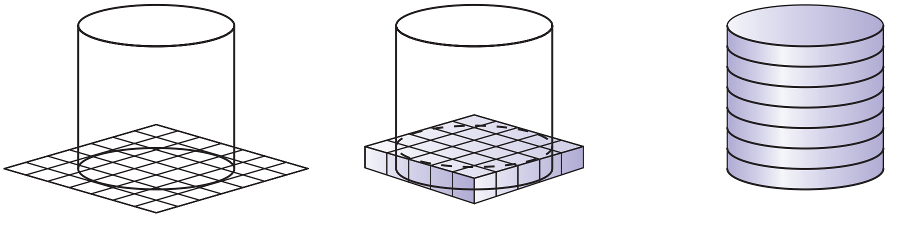
The last problem revealed some interesting connections among volume and surface area of prisms. A cylinder resembles a many-sided prism. In this problem you will explore cylinders and use what you have already learned about prisms to find the volume and surface area of a cylinder.

Directions for Making Paper Cylinders

- Start with two identical sheets of paper.
- Use the longer dimension of one sheet of paper as the height of the first cylinder. Tape the paper into the shape of a cylinder.
- Use the shorter dimension of the other sheet of paper as the height of the second cylinder. Tape the paper into the shape of a cylinder.

How do the volumes of the two cylinders compare?

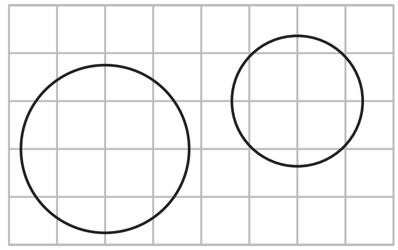
You need an efficient way to compute the volume of a cylinder. In Problem 3.1 you found the volume of a prism by counting the number of cubes that fit in a single layer at the base and then counting the number of layers it would take to fill the prism. Let's see if a similar approach will work for cylinders.



Trace the base. How many cubes would fit in one layer? How many layers would it take to fill the cylinder?

As with rectangular prisms, the bottom of any prism or cylinder is called the base.

Problem 3.2 Finding the Volumes of Cylinders



A. Copy the circles at the right onto inch graph paper. With two identical sheets of paper, make two models of cylinders with open tops and bases that match the bases drawn on the grid paper.

B. Predict which of the two cylinders has the greater volume.

C. **1.** How many inch cubes fit on the bottom layer of each cylinder?

2. How many layers of inch cubes are needed to fill each cylinder?

3. What is the total number of inch cubes needed to fill each cylinder?

4. How can the dimensions help you calculate the volume of each cylinder?

D. Suppose Cylinder 1 has a height of 10 centimeters and a radius of 4 centimeters and Cylinder 2 has a height of 4 centimeters and a radius of 10 centimeters. Are the volumes equal? Explain.

E. Suppose that each of your paper cylinders had a top and a bottom. Describe how you could find the surface area of each cylinder.

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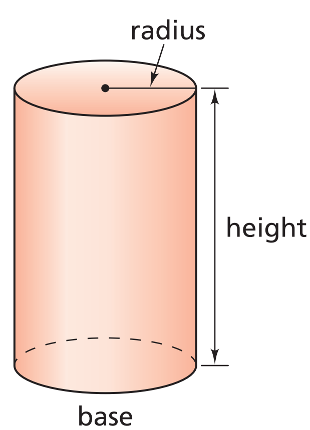
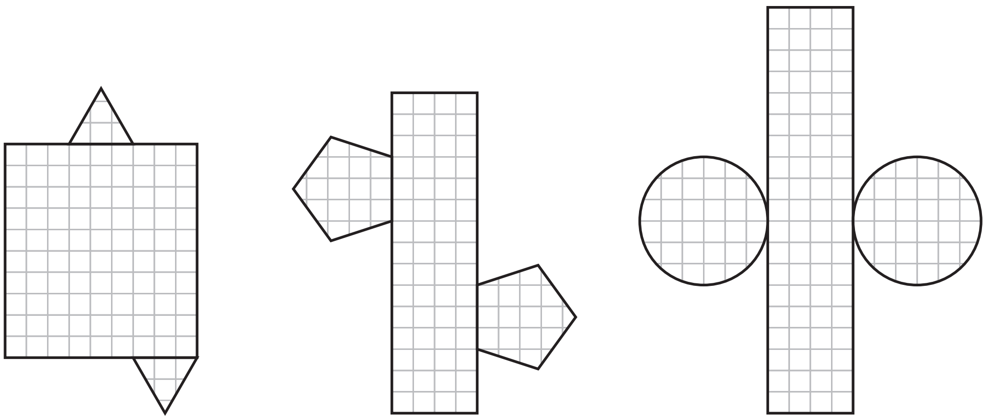
3.3 Making Cylinders and Prisms from Nets

The distance from the base of a cylinder to the top is called the height. You can describe a cylinder by giving its dimensions.



Problem 3.3 Finding the Surface Area of Cylinders and Prisms

Draw nets like the following on centimeter grid paper.



A. What is the surface area of each shape? Explain your reasoning.

B. Cut out your nets. Tape the pieces of the nets together to form a cylinder or a prism.

1. Describe how to find the surface area of any prism or cylinder.

2. Describe how the dimensions of a cylinder help you to find its surface area.

C. **1.** Find the volume of each prism and cylinder.

2. Compare the methods for finding the volume of a prism and finding the volume of a cylinder.

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Did You Know?

The volume, or capacity, of a liquid container is often given in units like quarts, gallons, liters, and milliliters. These volumes do not tell you how many unit cubes each container will hold, but are based on cubic measures. For example, a gallon equals 231 cubic inches, a milliliter equals a cubic centimeter, and a liter is 1,000 cubic centimeters.

3.4 Making a New Juice Container

Fruit Tree Juice Company packages its most popular drink, apple-prune juice, in cylindrical cans. Each can is 8 centimeters high and has a radius of 2 centimeters.

Recent reports indicate a decline in the sales of Fruit Tree juice. At the same time, sales of juice sold by a competitor, the Wrinkled Prune Company, are on the rise. Market researchers at Fruit Tree determine that Wrinkled Prune's success is due to its new rectangular juice boxes. Fruit Tree decides to package its juice in rectangular boxes.

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Problem 3.4 Comparing Volumes

Fruit Tree wants the new rectangular box to have the same volume as the current cylindrical can.

A. 1. On centimeter grid paper, make a net for a box that will hold the same amount of juice as the cylindrical can. Cut out your net. When you are finished, fold and tape your pattern to form a rectangular box.

2. Give the dimensions of your juice box. Are there other possibilities for the dimensions? Explain.

3. Compare your juice box with the boxes made by your classmates. Which rectangular box shape do you think would make the best juice container? Why?

B. Compare the surface area of the cylindrical can with the surface area of your juice box. Which container has greater surface area?

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