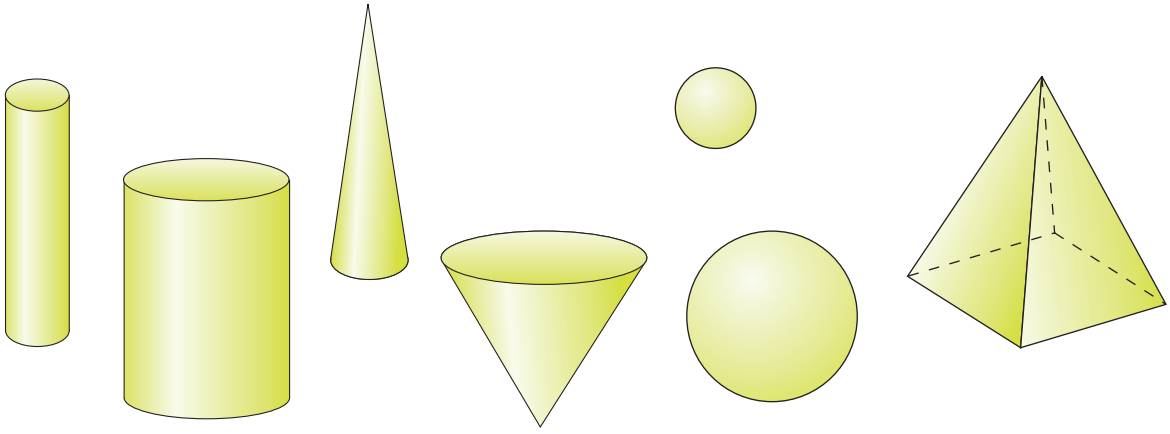
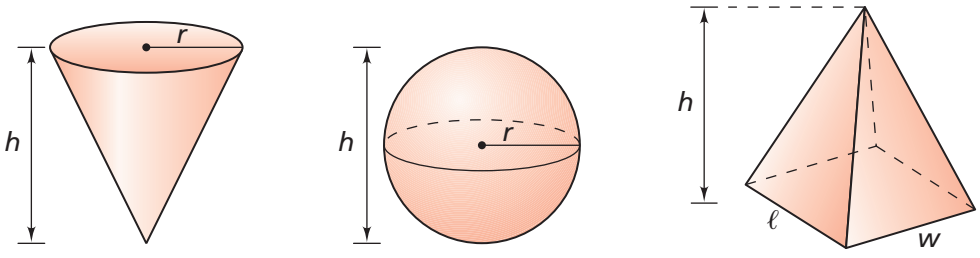
Investigation 4

Cones, Spheres, and Pyramids

Many common and important three-dimensional objects are not shaped like prisms or cylinders. For example, ice cream is often served in **cones.** The planet we live on is nearly a **sphere.** Many monuments here and in other countries are shaped like **pyramids.**



As with a cylinder and a prism, you can describe a cone or a square pyramid by giving its dimensions. The dimensions of a cone are the radius of its circular base and its height. The dimensions of a square pyramid are its length, width, and height.



Cone **Sphere** **Square Pyramid**

Although spheres may differ in size, they are all the same shape. You can describe a sphere by giving its radius.

In this investigation, you will explore ways to determine the volumes of cones, pyramids, and spheres by looking for relationships between cones and pyramids and between cones and spheres.

4.1 Comparing Spheres and Cylinders

In this problem, you will make a sphere and a cylinder with the same diameter and the same height and then compare their volumes. (The height of a sphere is just its diameter.) You can use the relationship you observe to develop a method for finding the volume of a sphere.

Did You Know?

Earth is nearly a sphere. You may have heard that, until Christopher Columbus's voyage in 1492, most people believed Earth was flat. Actually, as early as the fourth century B.C., scientists had figured out that Earth was round.

The scientists observed the shadow of Earth as it passed across the moon during a lunar eclipse. The shadow was round. Combining this observation with evidence gathered from observing constellations, these scientists concluded that Earth was spherical. In the third century B.C., Eratosthenes, a Greek mathematician, was actually able to estimate the circumference of Earth.

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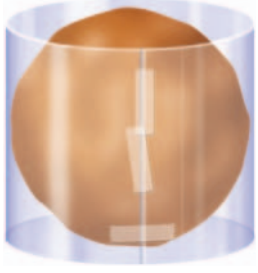
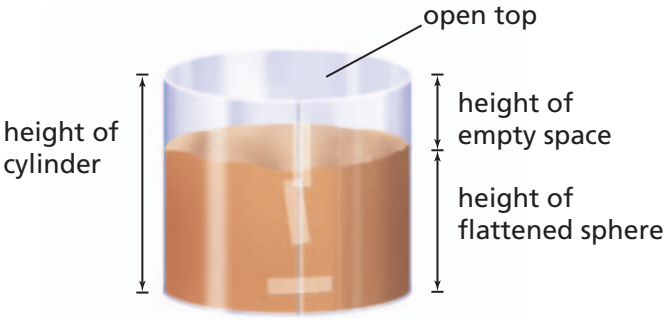
Problem 4.1 Comparing Spheres and Cylinders

- Make a sphere from modeling clay. Measure its diameter.
- Make a cylinder with an open top and bottom from a sheet of stiff transparent plastic to fit snugly around your clay sphere. Trim the height of the cylinder to match the height of the sphere. This makes the diameter and the height of the cylinder equal to the diameter and the height of the sphere. Tape the cylinder together so that it remains rigid.



- Now, flatten the clay sphere so that it fits snugly in the bottom of the cylinder. Mark the height of the flattened sphere on the cylinder.



A. Measure and record the height of the cylinder, the height of the empty space, and the height of the flattened sphere. Use this information to find the volume of the cylinder and the original sphere.

B. What is the relationship between the volume of the sphere and the volume of the cylinder?

C. A cylinder with a height equal to its diameter has a volume of 48 cubic inches. How can you use the relationship in Question B to find the volume of a sphere whose radius is the same as the cylinder?

Remove the clay from the cylinder and save the cylinder for Problem 4.2.

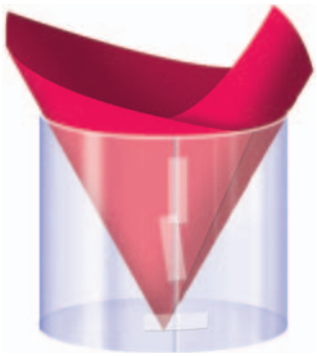
ACE **Homework starts on page 54.**

4.2 Cones and Cylinders, Pyramids and Cubes

In Problem 4.1, you discovered the relationship between the volume of a sphere and the volume of a cylinder. In this problem, you will look for the relationship between the volume of a cone and the volume of a cylinder, and between the volume of a pyramid and the volume of a square prism.

Problem 4.2 Cones and Cylinders, Pyramids and Cubes

- Roll a piece of stiff paper into a cone shape so that the tip touches the bottom of the cylinder you made in Problem 4.1.



- Tape the cone shape along the seam. Trim the cone so that it is the same height as the cylinder.

- Fill the cone to the top with sand or rice, and empty the contents into the cylinder. Repeat this as many times as needed to fill the cylinder completely.

A. What is the relationship between the volume of the cone and the volume of the cylinder?

B. Suppose a cylinder, a cone, and a sphere have the same radius and the same height. What is the relationship between the volumes of the three shapes?

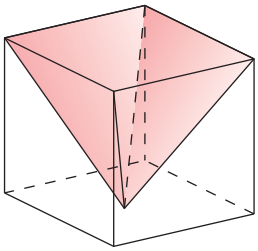
C. Suppose a cone, a cylinder, and a sphere all have the same height, and that the cylinder has a volume of 64 cubic inches. How do you use the relationship in Question B to find

1. the volume of a sphere whose radius is the same as the cylinder?

2. the volume of a cone whose radius is the same as the cylinder?

D. Suppose the radius of a cylinder, a cone, and a sphere is 5 centimeters and the height of the cylinder and cone is 8 centimeters. Find the volume of the cylinder, cone, and sphere.

E. 1. Use a square prism and a pyramid to conduct an experiment similar to the one on the previous page. The pyramid should have the same size base as the prism and the same height (shown at the right).



What is the relationship between the volume of the prism and the volume of the pyramid.

2. How are finding the volumes of the cones and pyramids alike?

ACE **Homework starts on page 54.**

4.3 Melting Ice Cream

Esther and Jasmine buy ice cream from Chilly's Ice Cream Parlor. They want to bring back an ice cream cone to Esther's little brother but decide the ice cream would melt before they got back home. Jasmine wonders, "If the ice cream all melts into the cone, will it fill the cone?"

Problem 4.3 Comparing Volumes of Spheres, Cylinders, and Cones

Esther gets a scoop of ice cream in a cone, and Jasmine gets a scoop in a cylindrical cup. Each container has a height of 8 centimeters and a radius of 4 centimeters. Each scoop of ice cream is a sphere with a radius of 4 centimeters.

A. Suppose Jasmine allows her ice cream to melt. Will the melted ice cream fill her cup exactly? Explain.

B. Suppose Esther allows her ice cream to melt. Will the melted ice cream fill her cone exactly? Explain.

C. How many same-sized scoops of ice cream of the size shown on the previous page can be packed into each container?

ACE **Homework starts on page 54.**

Frank and Ernest

Did You Know?

You have looked at prisms, cylinders, cones, and spheres. Many three-dimensional objects do not have such regular shapes.

According to legend, Archimedes (ahr kuh MEE deez) made an important discovery while taking a bath in the third century B.C. He noticed that the water level rose when he sat down in a tub. This was because his body had *displaced* some water. He determined that he could find the weight of any floating object by finding the weight of the water that the object displaced.

It is said that Archimedes was so excited about his discovery that he jumped from his bath and, without dressing, ran into the streets shouting "Eureka!"

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