# Applications

1. A group of students conducts the bridge-thickness experiment with construction paper. Their results are shown in this table.

Bridge-Thickness Experiment

Thickness (layers)	1	2	3	4	5	6
Breaking Weight (pennies)	12	20	29	42	52	61

    a. Make a graph of the (*thickness, breaking weight*) data. Describe the relationship between thickness and breaking weight.

    b. Suppose it is possible to use half-layers of construction paper. What breaking weight would you predict for a bridge 3.5 layers thick? Explain.

    c. Predict the breaking weight for a construction-paper bridge 8 layers thick. Explain how you made your prediction.

2. The table shows the maximum weight a crane arm can lift at various distances from its cab. (See the diagram below.)
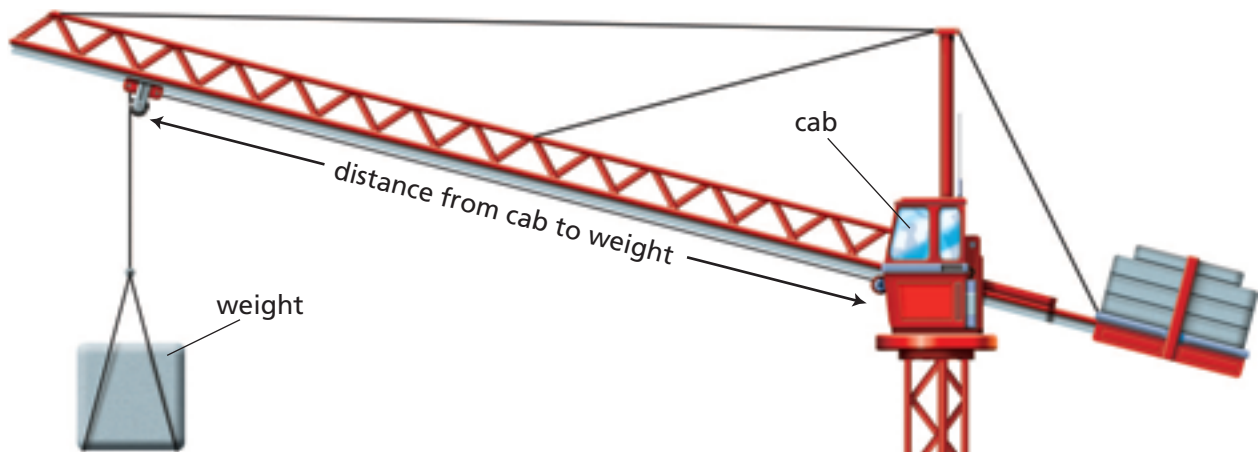
Construction-Crane Data

Distance from Cab to Weight (ft)	12	24	36	48	60
Weight (lb)	7,500	3,750	2,500	1,875	1,500

a. Describe the relationship between distance and weight for the crane.

b. Make a graph of the (*distance, weight*) data. Explain how the graph's shape shows the relationship you described in part (a).

c. Estimate the weight the crane can lift at distances of 18 feet, 30 feet, and 72 feet from the cab.

d. How, if at all, is the crane data similar to the data from the bridge experiments in Problems 1.1 and 1.2?

3. A beam or staircase frame from CSP costs $2.25 for each rod, plus $50 for shipping and handling.

a. Refer to your data for Question A of Problem 1.3. Copy and complete the following table to show the costs for beams of different lengths.

Costs of CSP Beams

Beam Length (ft)	1	2	3	4	5	6	7	8
Number of Rods	3	7	■	■	■	■	27	■
Cost of Beam	■	■	■	■	■	■	■	■

b. Make a graph of the (*beam length, cost*) data.

c. Describe the relationship between beam length and cost.

d. Refer to your data for Question B of Problem 1.3. Copy and complete the following table to show the costs for staircase frames with different numbers of steps.

Costs of CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	■	■	■	■	■
Cost of Frame	■	■	■	■	■	■	■	■

e. Make a graph of the (*number of steps, cost*) data.

f. Describe the relationship between the number of steps and the cost.
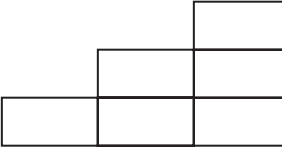
4. Parts (a)–(f) refer to relationships you have studied in this investigation. Tell whether each relationship is linear.

   a. the relationship between beam length and cost (ACE Exercise 3)

   b. the relationship between the number of steps in a staircase frame and the cost (ACE Exercise 3)

   c. the relationship between bridge thickness and strength (Problem 1.1)

   d. the relationship between bridge length and strength (Problem 1.2)

   e. the relationship between beam length and the number of rods (Problem 1.3)

   f. the relationship between the number of steps in a staircase frame and the number of rods (Problem 1.3)

   g. Compare the patterns of change for all the nonlinear relationships in parts (a)–(f).

5. In many athletic competitions, medals are awarded to top athletes. The medals are often awarded in ceremonies with medal winners standing on special platforms. The sketches show how to make platforms by stacking boxes.


1 medalist   2 medalists   3 medalists
1 box   3 boxes   6 boxes

a. Copy and complete the table below.

Medal Platforms

Number of Medalists	1	2	3	4	5	6	7	8
Number of Boxes	1	3	6	■	■	■	■	■

b. Make a graph of the (*number of medalists, number of boxes*) data.

c. Describe the pattern of change shown in the table and graph.

d. Each box is 1 foot high and 2 feet wide. A red carpet starts 10 feet from the base of the platform, and covers all the risers and steps.
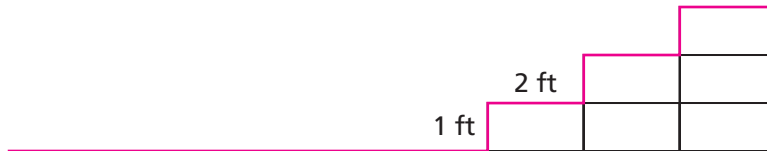

Copy and complete the table below.

Carpet for Platforms

Number of Steps	1	2	3	4	5	6	7	8
Carpet Length (ft)	■	■	■	■	■	■	■	■

e. Make a graph of the (*number of steps, carpet length*) data.

f. Describe the pattern of change in the carpet length as the number of steps increases. Compare this pattern with the pattern in the (*number of medalists, number of boxes*) data.

6. CSP also sells ladder bridges made from 1-foot steel rods arranged to form a row of squares. Below is a 6-foot ladder bridge.
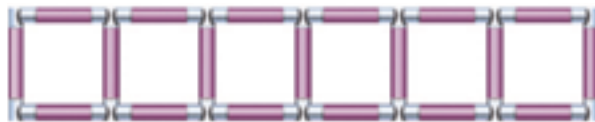

6-foot ladder bridge made from 19 rods

a. Make a table and a graph showing how the number of rods in a ladder bridge is related to length of the bridge.

b. Compare the pattern of change for the ladder bridges with those for the beams and staircase frames in Problem 1.3.

Connections

A survey of one class at Pioneer Middle School finds that 20 out of 30 students would spend $8 for a school T-shirt. Use this information for Exercises 7 and 8.

7. **Multiple Choice** Suppose there are 600 students in the school. Based on the survey, how many students do you predict would spend $8 for a school T-shirt?

    A. 20   **B.** 200

    C. 300   **D.** 400

8. **Multiple Choice** Suppose there are 450 students in the school. Based on the survey, how many students do you predict would spend $8 for a school T-shirt?

    F. 20   **G.** 200

    H. 300   **J.** 400

9. Below is a drawing of a rectangle with an area of 300 square feet.

    20 ft

    15 ft
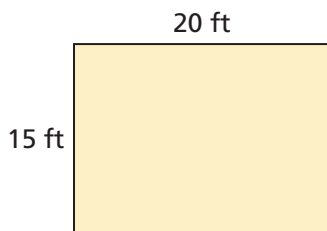
    a. Make drawings of at least three other rectangles with an area of 300 square feet.

    b. What is the width of a rectangle with an area of 300 square feet if its length is 1 foot? If its length is 2 feet? If its length is 3 feet?

    c. What is the width of a rectangle with an area of 300 square feet and a length of *L* feet?

    d. How does the width of a rectangle change if the length increases, but the area remains 300 square feet?

    e. Make a graph of (*width, length*) pairs for a rectangle that give an area of 300 square feet. Explain how your graph illustrates your answer for part (d).

10. a. The rectangle pictured in Exercise 9 has a perimeter of 70 feet. Make drawings of at least three other rectangles with a perimeter of 70 feet.

b. What is the width of a rectangle with a perimeter of 70 feet if its length is 1 foot? 2 feet? L feet?

c. What is the width of a rectangle with a perimeter of 70 feet if its length is $\frac{1}{2}$ foot? $\frac{3}{2}$ feet?

d. Give the dimensions of rectangles with perimeters of 70 feet and length-to-width ratios of 3 to 4, 4 to 5, and 1 to 1.

e. Suppose the length of a rectangle increases, but the perimeter remains at 70 feet. How does the width change?

f. Make a graph of (*width, length*) pairs that give a perimeter of 70 feet. How does your graph illustrate your answer for part (e)?

11. The 24 students in Ms. Cleary's homeroom are surveyed. They are asked which of several prices they would pay for a ticket to the school fashion show. The results are shown in this table.

Ticket-Price Survey

Ticket Price	$1.00	$1.50	$2.00	$2.50	$3.00	$3.50	$4.00	$4.50
Probable Sales	20	20	18	15	12	10	8	7

a. There are 480 students in the school. Use the data from Ms. Cleary's class to predict ticket sales for the entire school for each price.

b. Use your results from part (a). For each price, find the school's projected income from ticket sales.

c. Which price should the school charge if it wants to earn the maximum possible income?

ADMIT ONE

Middletown High School Presents:
The Spring Fashion Show
Featuring
★ Mathematical Models ★
Friday, May 23, at 7:00 PM
Middletown High School Auditorium

Tell which graph matches the equation or the set of criteria.

12. $y = 3x + 1$

13. $y = -2x + 2$
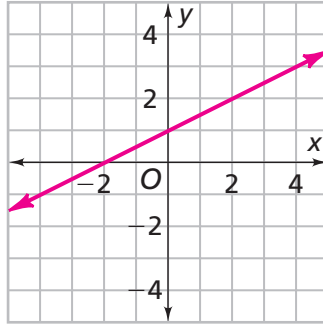
14. $y = x - 3$

15. y-intercept $= 1$; slope $= \frac{1}{2}$
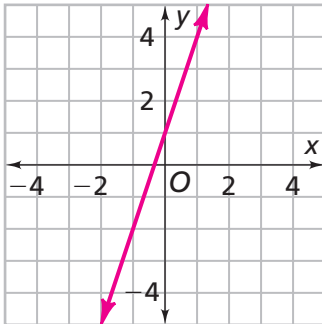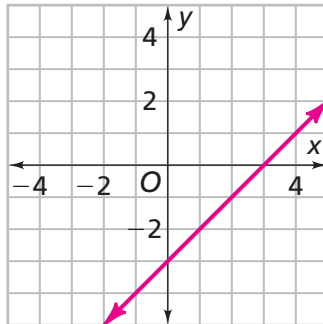
Graph A
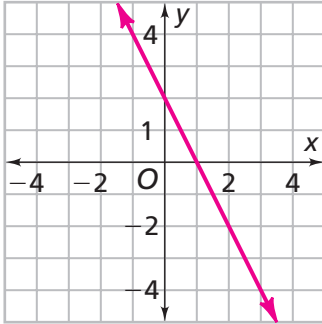
Graph B

Graph C

Graph D

Within each equation, the pouches shown contain the same number of coins. Find the number of coins in each pouch. Explain your method.

16.

17.

18. Refer to Exercises 16 and 17.

    a. For each exercise, write an equation to represent the situation. Let x represent the number of coins in a pouch.

    b. Solve each equation. Explain the steps in your solutions.

    c. Compare your strategies with those you used in Exercises 16 and 17.

Solve each equation for x.

19. $3x + 4 = 10$

20. $6x + 3 = 4x + 11$

21. $6x - 3 = 11$

22. $-3x + 5 = 7$

23. $4x - \frac{1}{2} = 8$

24. $\frac{x}{2} - 4 = -5$

25. $3x + 3 = -2x - 12$

26. $\frac{x}{4} - 4 = \frac{3x}{4} - 6$

For Exercises 27–29, tell whether the statement is _true_ or _false_. Explain your reasoning.
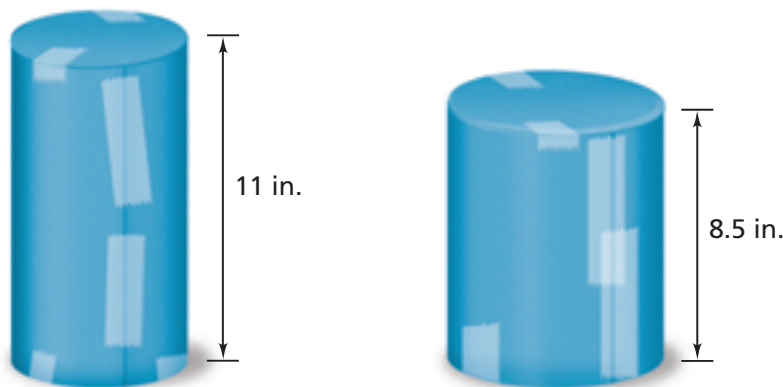
27. $6(12 - 5) > 50$

28. $3 \cdot 5 - 4 > 6$

29. $10 - 5 \cdot 4 > 0$

**30.** You will need two sheets of 8.5- by 11-inch paper and some scrap paper.

    a. Roll one sheet of paper to make a cylinder 11 inches high. Overlap the edges very slightly and tape them together. Make bases for the cylinder by tracing the circles on the ends of the cylinder, cutting out the tracings, and taping them in place.

11 in.

8.5 in.

    b. Roll the other sheet of paper to make a cylinder 8.5 inches high. Make bases as you did in part (a).

    c. Do the cylinders appear to have the same surface area (including the bases)? If not, which has the greater surface area?

    d. Suppose you start with two identical rectangular sheets of paper which are _not_ 8.5 by 11 inches. You make two cylinders as you did before. Which cylinder will have the greater surface area, the taller cylinder or the shorter one? How do you know?

Connections

31. The volume of the cone in the drawing at right is $\frac{1}{3}(28)\pi$. What are some possible radius and height measurements for the cone?
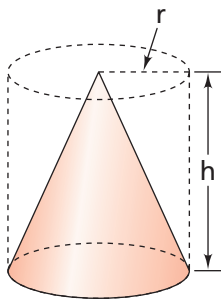
Extensions

32. Study the patterns in this table. Note that the numbers in the x column may not be consecutive after $x = 6$.

x	p	q	y	z
1	1	1	2	1
2	4	8	4	$\frac{1}{2}$
3	9	27	8	$\frac{1}{3}$
4	16	64	16	$\frac{1}{4}$
5	25	125	32	$\frac{1}{5}$
6	■	■	■	■
■	■	■	1,024	■
■	■	■	2,048	■
■	■	1,728	■	■
n	■	■	■	■

a. Use the patterns in the first several rows to find the missing values.

b. Are any of the patterns linear? Explain.

**33.** The table gives data for a group of middle school students.

Data for Middle School Students

| Student	Name Length	Height (cm)	Foot Length (cm)
Thomas Petes	11	126	23
Michelle Hughes	14	117	21
Shoshana White	13	112	17
Deborah Locke	12	127	21
Tonya Stewart	12	172	32
Richard Mudd	11	135	22
Tony Tung	8	130	20
Janice Vick	10	134	21
Bobby King	9	156	29
Kathleen Boylan	14	164	28

a. Make a graph of the (*name length, height*) data, a graph of the (*name length, foot length*) data, and a graph of the (*height, foot length*) data.

b. Look at the graphs you made in part (a). Which seem to show linear relationships? Explain.

c. Estimate the average height-to-foot-length ratio. That is, how many "feet" tall is the typical student in the table?

d. Which student has the greatest height-to-foot-length ratio? Which student has the least height-to-foot-length ratio?

34. A staircase is a prism. This is easier to see if the staircase is viewed from a different perspective. In the prism below, the small squares on the top each have an area of 1 square unit.

Top

Right

Bottom
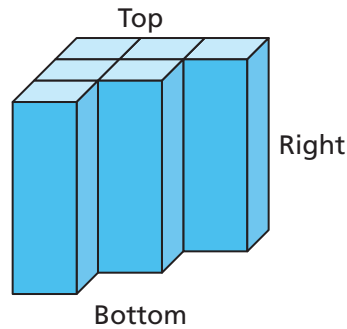
a. Sketch the base of the prism. What is the area of the base?

b. Rashid is trying to draw a *net* (flat pattern) that will fold up to form the staircase prism. Below is the start of his drawing. Finish Rashid's drawing and give the surface area of the entire staircase. **Hint:** You may want to draw your net on grid paper and then cut it out and fold it to check.

Left   Rear
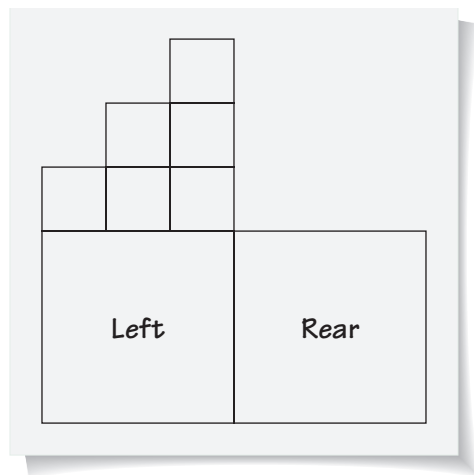
c. Suppose the prism had six stairs instead of three. Assume each stair is the same width as those in the prism above. Is the surface area of this six-stair prism twice that of the three-stair prism? Explain.