Investigation 2

Linear Models and Equations

Organizing and displaying the data from an experiment or survey can help you spot trends and make predictions. When the data show a linear trend, you can find a graph and equation to *model* the relationship between the variables. You can then use the model to make predictions about values between and beyond the data values.

When you make a model to represent a mathematical relationship, examine your model and ask

For what interval of values is the model likely to be reasonably accurate?

2.1 Linear Models

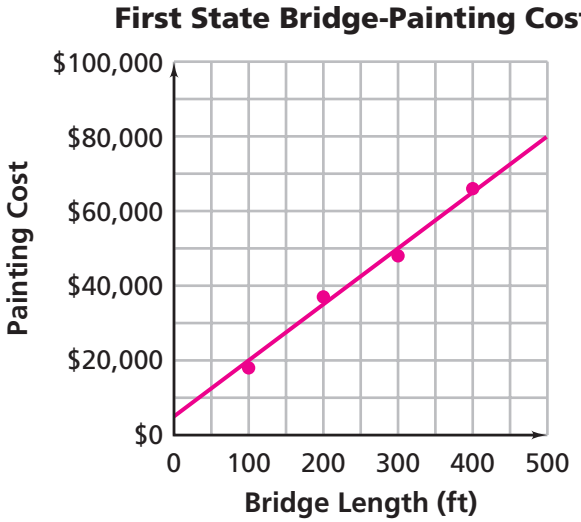
The First State Bridge-Painting Company is often asked to bid on painting projects. It usually gets the contract if it offers the lowest price. However, it needs to make sure the bid is high enough that the company will make a reasonable profit.

First State is preparing a bid for a bridge-painting project. The company looks at its records for previous projects. It finds information about four bridges with similar designs.

First State Bridge-Painting Costs

Bridge Number	Length (ft)	Painting Cost
1	100	$18,000
2	200	$37,000
3	300	$48,000
4	400	$66,000

The First State cost estimators plot the data. The points fall in a nearly linear pattern. They draw a line that fits the pattern well. The line is a **mathematical model** for the relationship between bridge length and painting cost. A mathematical model approximates a data pattern.



Getting Ready for Problem 2.1

A mathematical model can be used to make predictions about values between and beyond the data points.

- How do you think the cost estimators decided where to draw the line?
- Is the line a reasonable model for these data?
- What information does the model give that the points alone do not?
- What questions could you answer using the model?
- What information do you need to write an equation for the line?

Problem 2.1 Linear Models

A. 1. Write an equation for the line that models the data.

2. Use the line or the equation to estimate painting costs for similar bridges that are

a. 175 feet long **b.** 280 feet long

3. Use the line or the equation to estimate lengths of similar bridges for which the painting costs are

a. $10,000 **b.** $60,000

B. First State is also bidding on a different type of bridge. It has records for three similar bridges.

First State Bridge-Painting Costs

Bridge Number	Length (ft)	Painting Cost
3	150	$50,000
4	300	$80,000
5	500	$140,000

1. Plot these data points. Draw a line that models the pattern in the data points.
2. Write an equation for your line.
3. Use your equation or line to estimate the painting cost for a similar bridge that is 200 feet long.
4. Use your equation or line to estimate the length of a similar bridge that costs $100,000 to paint.

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2.2 Equations for Linear Relationships

Cars and trucks are an important part of American life and culture. There are nearly 200 million licensed drivers and 140 million registered passenger cars in the United States. To help people keep their cars clean, many cities have self-service car washes.

At most self-service car washes, the charge for washing a car and the company's profit depend on the time the customer spends using the car wash. To run such a business efficiently, it helps to have equations relating these key variables.

Getting Ready for Problem 2.2

- Sudzo Wash and Wax charges customers \$0.75 per minute to wash a car. Write an equation that relates the total charge c to the amount of time t in minutes.
- Pat's Power Wash charges \$2.00 per car to cover the cost of cleaning supplies, plus \$0.49 per minute for the use of water sprayers and vacuums. Write an equation for the total charge c for any car-wash time t.
- U-Wash-It charges \$10 for each car. The business owners estimate that it costs them \$0.60 per minute to provide soap, water, and vacuums for a car. Write an equation for the profit p U-Wash-It earns if a customer spends t minutes washing a car.
- Explain what the numbers and variables in each equation represent.
- What questions can your equations help you answer?

Problem 2.2 Equations for Linear Relationships

A. The Squeaky Clean Car Wash charges by the minute. This table shows the charges for several different times.

Squeaky Clean Car Wash Charges

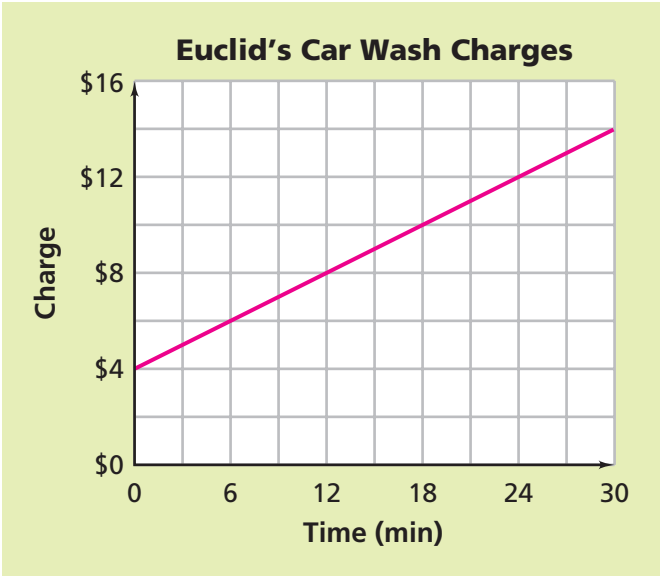
Time (min)	5	10	15	20	25
Charge	$8	$13	$18	$23	$28

1. Explain how you know the relationship is linear.

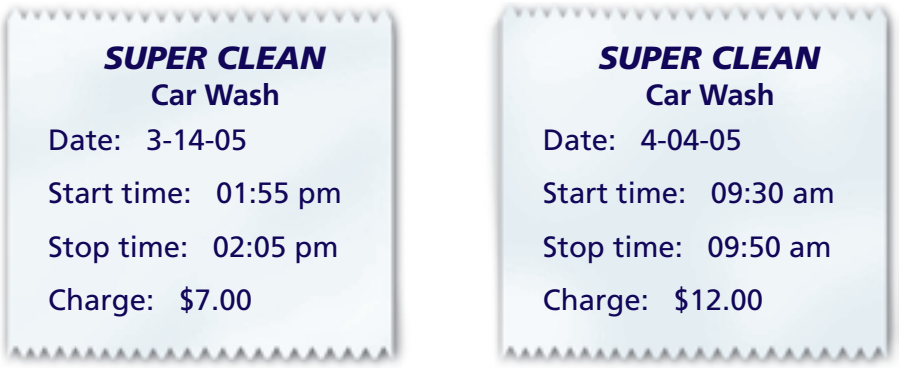
2. What are the slope and y-intercept of the line that represents the data?

3. Write an equation relating charge c to time t in minutes.

B. Euclid's Car Wash displays its charges as a graph. Write an equation for the charge plan at Euclid's. Describe what the variables and numbers in your equation tell you about the situation.



C. Below are two receipts from Super Clean Car Wash. Assume the relationship between charge c and time used t is linear.



1. Each receipt represents a point (t, c) on the line. Find the coordinates of the two points.
2. What are the slope and y-intercept of the line?
3. Write an equation relating c and t.

D. Write an equation for the line with slope -3 that passes through the point $(4, 3)$.

E. Write an equation for the line with points $(4, 5)$ and $(6, 9)$.

F. Suppose you want to write an equation of the form $y = mx + b$ to represent a linear relationship. What is your strategy if you are given

1. a description of the relationship in words?
2. two or more (x, y) values or a table of (x, y) values?
3. a graph showing points with coordinates?

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2.3 Solving Linear Equations

Sandy's Boat House rents canoes. The equation $c = 0.15t + 2.50$ gives the charge c in dollars for renting a canoe for t minutes.

Getting Ready for Problem 2.3

- Explain what the numbers in the equation $c = 0.15t + 2.50$ tell you about the situation.
- Rashida and Serena apply for jobs at Sandy's. The manager tests them with three questions.

  What is the charge for renting a canoe for 30 minutes?

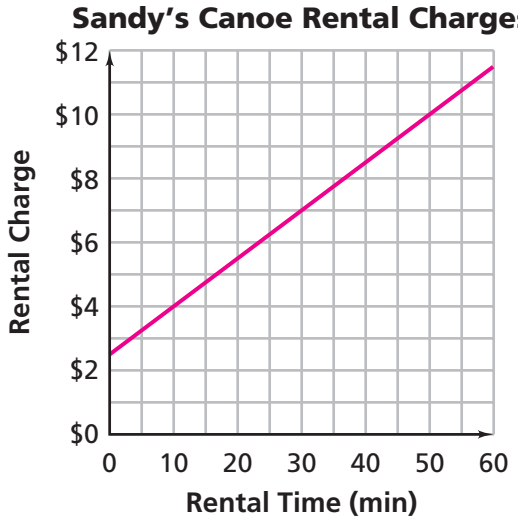
  A customer is charged $8.50. How long did he use the canoe?

  A customer has $10 to spend. How long can she use a canoe?

  Suppose you were applying for a job at Sandy's. How would you answer these questions?

Problem 2.3 Solving Linear Equations

A. Rashida uses a graph of $c = 0.15t + 2.50$. Explain how to use the graph to estimate the answers to the manager's questions.



B. Rashida could use a table instead of a graph. Explain how to use a table to estimate answers to the questions.

C. Serena wants to find exact answers, not estimates. For the second question, she solves the linear equation $0.15t + 2.50 = 8.50$. She reasons as follows:

- If $0.15t + 2.50 = 8.50$, then $0.15t = 6.00$.
- If $0.15t = 6.00$, then $t = 40$.
- I check my answer by substituting 40 for t: $0.15(40) + 2.50 = 8.50$

Is Serena correct? How do you know?

D. For the third question, Rashida says, "She can use the canoe for 50 minutes if she has \$10." Serena says there are other possibilities—for example, 45 minutes or 30 minutes. She says you can answer the question by solving the **inequality** $0.15t + 2.50 \leq 10$. This inequality represents the times for which the rental charge is *at most* \$10.

1. Use a table, a graph, and the equation $0.15t + 2.50 = 10$ to find all of the times for which the inequality is true.
2. Express the solution as an inequality.

E. River Fun Paddle Boats competes with Sandy's. The equation $c = 4 + 0.10t$ gives the charge in dollars c for renting a paddle boat for t minutes.

1. A customer at River Fun is charged \$9. How long did the customer use a paddle boat? Explain.
2. Suppose you want to spend \$12 at most. How long could you use a paddle boat? Explain.
3. What is the charge to rent a paddle boat for 20 minutes? Explain.

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2.4 Intersecting Linear Models

A resort area has two main attractions—the Big Fun amusement park and the Get Reel movie multiplex. The number of visitors to each attraction on a given day is related to the probability of rain.

This table gives attendance and rain-forecast data for several Saturdays.

Saturday Resort Attendance

Probability of Rain (%)	0	20	40	60	80	100
Big Fun Attendance	1,000	850	700	550	400	250
Get Reel Attendance	300	340	380	420	460	500

The same company owns both businesses. The managers want to be able to predict Saturday attendance at each attraction so they can assign their workers efficiently.

Problem 2.4 Intersecting Linear Models

A. Use the table to find a linear equation relating the probability of rain p to

1. Saturday attendance A_B at Big Fun.

2. Saturday attendance A_G at Get Reel.

B. Use your equations from Question A to answer these questions. Show your calculations and explain your reasoning.

1. Suppose there is a 50% probability of rain this Saturday. What is the expected attendance at each attraction?

2. Suppose 460 people visited Big Fun one Saturday. Estimate the probability of rain on that day.

3. What probability of rain would give a predicted Saturday attendance of at least 360 people at Get Reel?

4. Is there a probability of rain for which the predicted attendance is the same at both attractions? Explain.

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