Investigation 3

Inverse Variation

In Investigation 1, you discovered that the relationship between bridge thickness and bridge strength is approximately linear. You also found that the relationship between bridge length and bridge strength is not linear. In this investigation, you will explore other nonlinear relationships.

3.1 Rectangles With Fixed Area

In recent years, the populations of many small towns have declined as residents move to large cities for jobs. The town of Roseville has developed a plan to attract new residents. The town is offering free lots of land to "homesteaders" who are willing to build houses. Each lot is rectangular and has an area of 21,800 square feet. The lengths and widths of the lots vary.

Getting Ready for Problem 3.1

- What are some possible dimensions for a rectangular lot with an area of 21,800 square feet?

In Problem 3.1, you will look at patterns in length and width values for rectangles with fixed area.

Problem 3.1 Relating Length and Width

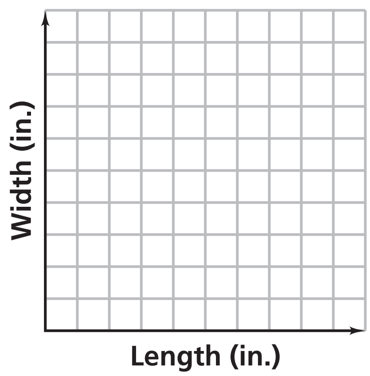
A. 1. Copy and complete this table.

Rectangles With Area 24 in.^{2}

Length (in.)	1	2	3	4	5	6	7	8
Width (in.)	■	■	■	■	■	■	■	■

2. Plot your data on a grid like the one below. Then, draw a line or curve that seems to model the pattern in the data.

Rectangles With Area 24 in.^{2}



3. Describe the pattern of change in the width as the length increases. Is the relationship between length and width linear?

4. Write an equation that shows how the width w depends on the length ℓ for rectangles with an area of 24 square inches.

B. Now consider rectangles with an area of 32 square inches.

1. Write an equation for the relationship between the length ℓ and the width w.

2. Graph your equation. Show lengths from 1 to 15 inches.

C. Compare your equations. How are they similar? How are they different?

D. Compare your graphs. How are they similar? How are they different?

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3.2 Bridging the Distance

The relationship between length and width for rectangles with a fixed area is not linear. It is an example of an important type of nonlinear pattern called an **inverse variation.**

The word "inverse" suggests that as one variable increases in value, the other variable decreases in value. However, the meaning of *inverse variation* is more specific than this. The relationship between two non-zero variables, x and y, is an inverse variation if

$$y = \frac{k}{x}, \text{ or } xy = k$$

where k is a constant that is not 0. The value of k is determined by the specific relationship.

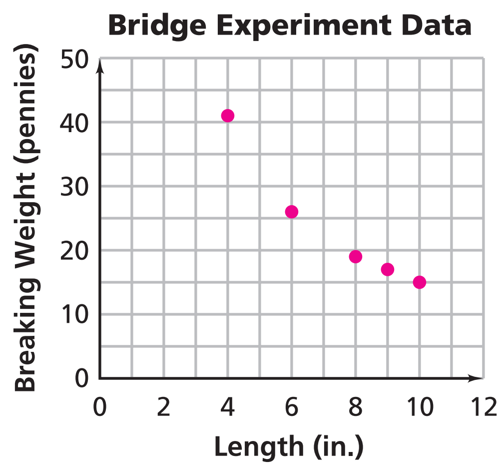
How are the equations $y = \frac{k}{x}$ *and* $xy = k$ *related?*

For the same x*-value, will the two equations give different* y*-values?*

Inverse variation occurs in many situations. For example, consider the table and graph below. They show the (*bridge length, breaking weight*) data collected by a group of students.

Bridge Experiment Data

Length (in.)	Breaking Weight (pennies)
4	41
6	26
8	19
9	17
10	15

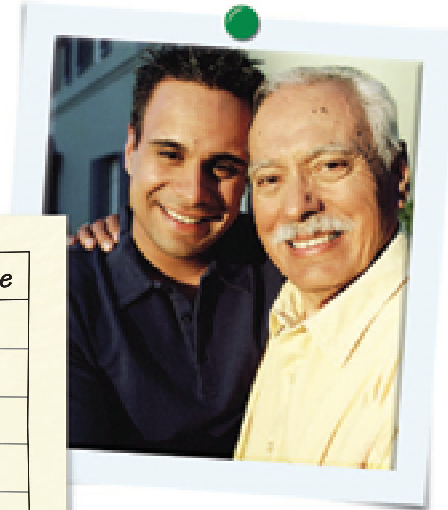


Getting Ready for Problem 3.2

- Describe a curve that models the pattern in the data above.
- What value of k can you use to model these data with an inverse variation equation? Write the equation.
- In your equation, why does the value of y decrease as the value of x increases?
- What happens to the value of y as the value of x gets close to 0? Why is that a reasonable pattern for the bridge experiment?

Problem 3.2 Inverse Variation Patterns

Mr. Cordova lives in Detroit, Michigan. He often travels to Baltimore, Maryland, to visit his grandfather. The trip is 500 miles each way. Here are his notes for his trips to Baltimore last year.



Date	Notes	Travel Time
February 15	Traveled by plane.	1.5 hours
May 22	Drove.	10 hours
July 3	Drove. Stopped for repairs.	14 hours
November 23	Flew. Flight delayed.	4 hours
December 23	Took overnight train.	18 hours

A. 1. Calculate the average speed in miles per hour for each trip. Record the results in a table like this.

Cordova's Baltimore Trips

Travel Time (hr)	■	■	■	■	■
Average Speed (mph)	■	■	■	■	■

2. Plot the data. Draw a line or curve that models the data pattern. Describe the pattern of change in average speed as travel time increases.

3. Write an equation for the relationship between travel time t and average speed s.

4. Use your equation to find the average speed for 500-mile trips that take 6 hours, 8 hours, 12 hours, and 16 hours.

5. Add the (*travel time, average speed*) data from part (4) to your graph. Do the new points fit the graph model you sketched for the original data?

B. The Cordova family is planning a trip to Mackinac Island (mak uh naw) near the upper peninsula of Michigan. Mr. Cordova does some calculations to see how the travel time will change if the family drives at different average speeds.

Travel Times for Different Speeds

Average Speed (mi/h)	30	40	50	60	70
Travel Time (hr)	10	7.5	6	5	4.3

1. How far is it from Detroit to Mackinac Island?
2. What equation relates travel time t to average speed s?
3. Describe the pattern of change in the travel time as the average speed increases. How would that pattern appear in a graph of the data? How is it shown by your equation?
4. Predict the travel times if the Cordovas drive at average speeds of 45 miles per hour and 65 miles per hour.

C. Suppose Mr. Cordova decides to aim for an average speed of 50 miles per hour for the trip to Mackinac Island.

1. Make a table and graph to show how the distance traveled will increase as time passes. Show times from when the family leaves home to when they reach their destination.
2. Write an equation for the distance d the family travels in t hours.
3. Describe the pattern of change in the distance as time passes.
4. Compare the (*time, distance traveled*) graph and equation with the (*time, average speed*) graphs and equations in Questions A and B.

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3.3 Average Cost

The science teachers at Everett Middle School want to take their eighth-graders on an overnight field trip to a nature center. It costs \$750 to rent the center facilities. The school budget does not provide funds to rent the nature center, so students must pay a fee. The trip will cost \$3 per student if all 250 students go. However, the teachers know it is unlikely that all students can go. They want to find the cost per student for any number of students.

Problem 3.3 Inverse Variation Patterns

A. **1.** Write an equation relating the cost c per student to the number of students n.

2. Use your equation to make a graph showing how the cost per student changes as the number of students increases.

B. **1.** Find the change in the cost per student as the number of students increases from

a. 10 to 20 **b.** 100 to 110 **c.** 200 to 210

2. How do your results show that the relationship between the number of students and the cost per student is not linear?

C. **1.** Find the change in the per-student cost as the number of students increases from

a. 20 to 40 **b.** 40 to 80 **c.** 80 to 160

2. Describe the pattern in your results. Explain how your equation from Question A shows this pattern.

D. The science teachers decide to charge \$5 per student for the trip. They will use any extra money to buy science equipment for the school.

1. Write an equation for the amount a the teachers will collect if n students go on the trip.

2. Sketch a graph of the relationship.

3. Is this a linear relationship or an inverse variation? Explain.

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