7.1 – Polynomial Degree and Finite Differences

Objectives:

1. Define polynomial, monomial, binomial, and trinomial
2. Determine degree by finite differences
3. Write polynomials in general form
4. Use finite differences and systems of equations to find a polynomial function that fits a data set.

Definition of a Polynomial

A polynomial in one variable is any expression that can be written in the form

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0$$

where x is a variable, the exponents are nonnegative integers, the coefficients are real numbers, and $a_n \neq 0$. Each monomial being added to make the polynomial is a term.

Polynomial Function: A function in which a polynomial __expression__ is set equal to a second variable.

Degree: In a __one__-variable polynomial, the power of the term that has the greatest exponent.

In a __multi-__variable polynomial, the greatest __sum__ of the powers in a single term.

General Form: The form of a polynomial in which the terms are ordered such that the degrees of the terms __decrease__ from left to right.

Monomial: A polynomial with __one__ term.

Binomial: A polynomial with __two__ terms.

Trinomial: A polynomial with __three__ terms.

Example 1: Find the difference of the y-values until the differences are constant. What do you notice??

$y = 3x + 4$
Degree: 1

X	Y
2	10
3	13
4	16
5	19
6	22
7	25

+3, +3, +3, +3, +3

$y = 2x^2 - 5x - 7$
Degree: 2

X	Y
3.7	1.88
3.8	2.88
3.9	3.92
4.0	5.00
4.1	6.12
4.2	7.28

+1, +1.04, +1.08, +.12, +1.16

+.04, +.04, +.04, +.04

$y = 0.1x^3 - x^2 + 3x - 5$
Degree: 3

X	Y
-5	-57.5
0	-5
5	-2.5
10	25
15	152.5
20	455

+52.5, +2.5, +27.5, +127.5, +302.5

-50, +25, +100, +175

+75, +75, +75

Finite Differences Method: A method of finding the degree of a polynomial that will model a set of data, by analyzing differences between data values corresponding to equally spaced values of the independent variable.

Example 2: Determine what degree the polynomials represented by the tables below will be.

X	Y
10	1018
11	1349
12	1746
13	2215
14	2762
15	3393

331, 397, 464, 547, 631

66, 72, 78, 84

6, 6, 6

Degree 3

X	Y
-5	-1250
0	0
5	-1250
10	-20,000
15	-101,250
20	-320,000

+1250, -1250, -18,750, -81,250, -218,750

-2500, -17,500, -62,500, -137,500

-15,000, -45,000, -75,000

-30,000, -30,000

Degree 4

Example 3: The table below represents the number of diagonals a polygon of n-sides has. Write a function to determine how many diagonals a 35-gon has.

a. Determine the degree of the function.

X [Number of sides]	Y [Number of diagonals]
3	0
4	2
5	5
6	9
7	14
8	20

$\rangle 2$ $\rangle 3$ $\rangle 4$ $\rangle 5$ $\rangle 6$

$\rangle 1$ $\rangle 1$ $\rangle 1$ $\rangle 1$

Because the function is quadratic, we can model the function as $y = ax^2 + bx + c$

Because we have 3 unknown values (a, b, c) in our function, we will need to write three equations.

Pick 3 points to help us write our equation – 1 point per equation. Substitute the x- and y-values into our general function:

$0 = a(3)^2 + b \cdot 3 + c$ $\quad 0 = 9a + 3b + c$

$2 = a(4)^2 + b \cdot 4 + c$ $\quad 2 = 16a + 4b + c$

$5 = a(5)^2 + b \cdot 5 + c$ $\quad 5 = 25a + 5b + c$

Solve the system above [substitution, elimination, or matrices]

$c = -9a - 3b$

$2 = 16a + 4b - 9a - 3b$

$5 = 25a + 5b - 9a - 3b$

$2 = 7a + b$

$5 = 16a + 2b$

$b = 2 - 7a$

$5 = 16a + 2(2 - 7a)$

$5 = 16a + 4 - 14a$

$5 = 2a + 4$

$1 = 2a$

$\frac{1}{2} = a$

$b = 2 - 7(\frac{1}{2})$

$b = 2 - 3.5$

$b = -1\frac{1}{2}$

$c = -9(\frac{1}{2}) - 3(-1\frac{1}{2})$

$= -4\frac{1}{2} + 4\frac{1}{2}$

$c = 0$

The function that represents our table: $y = \frac{1}{2}x^2 - \frac{3}{2}x + 0$

Use the equation above to determine how many diagonals a 35-gon has.

$y = \frac{1}{2}(35)^2 - \frac{3}{2}(35)$

$= \frac{1}{2}(1225) - 52\frac{1}{2}$

$= 612.5 - 52.5$

$= 560$ diagonals